Spring 2013 McNabb GDCTM Contest
Pre-Algebra

NO Calculators Allowed

1. If 10% of a is b what is 10% of b?

   (A) $100a$ **(B)** $10a$ **(C)** a **(D)** $.1a$ **(E)** $.01a$

2. If 10 carpenters can build 10 cabinets in 10 days how many days does it take 20 carpenters to build 20 cabinets?

   (A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

3. Express the fraction

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5}}}$$

   in lowest terms.

   (A) 1/8 **(B)** 11/15 **(C)** 15/21 **(D)** 16/21 **(E)** 21/16

4. The square root of 20000 lies between

   (A) 130 and 131 **(B)** 140 and 141 **(C)** 141 and 142
   (D) 142 and 143 **(E)** 10,000 and 10,001

5. The last 6 digits of 13^{426} are 000009. What is the sum of the last 6 digits of 13^{1704}?

   (A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

6. In the repeating decimal $0.\overline{71771}$, in which decimal place does the 2013th 7 appear?

   (A) 671st **(B)** 2014th **(C)** 2015th **(D)** 3354th **(E)** 3355th

7. How many positive factors does 2013 have?

   (A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

8. If the integer $\underline{4400b074}$ is divisible by 101, what must the digit b equal?

(A) 0 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 8

9. The points A, B, C, and D are the vertices of a unit square. How many squares in the plane of these points (including $ABCD$ itself) have two or more of them as vertices?

(A) 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 13

10. The number of digits in the large number 2^{50} is

(A) between 6 and 10 inclusive
(B) between 11 and 15 inclusive
(C) between 16 and 20 inclusive
(D) between 21 and 25 inclusive
(E) 26 or more

11. Four indistinguishable roses and two indistinguishable tulips are to be arranged in a circle. Two such arrangements are considered to be the same if and only if each can be rotated into the other. How many distinct arrangements are possible?

(A) 3 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 15

12. How many seconds are there in exactly six weeks?

(A) 7! **(B)** 8! **(C)** 9! **(D)** 10! **(E)** 12!

13. The product of a certain integer and 180 is a perfect square. That certain integer must be divisible by

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. A rectangle with area 125 has its sides in the ratio of 4 : 5. What is the perimeter of this rectangle?

(A) 18 **(B)** 22.5 **(C)** 36 **(D)** 45 **(E)** 54

15. I have two numbers in mind. The first number leaves a remainder of 4159 when divided by 5153 while the second number leaves a remainder of 5149 when divided by 5153. What is the remainder when the sum of these numbers is divided by 5153?

(A) 3135 **(B)** 3455 **(C)** 4144 **(D)** 4155 **(E)** 4344

16. In the following triangular arrangement of the positive integers, in which column, counting from left to right, does 7021 appear?

$$\begin{array}{cccc}
 & & & \vdots \\
 & & 5 & \cdots \\
 & 2 & 6 & \cdots \\
1 & 3 & 7 & \cdots \\
 & 4 & 8 & \cdots \\
 & & 9 & \cdots \\
 & & & \vdots
\end{array}$$

(A) 43 **(B)** 51 **(C)** 52 **(D)** 84 **(E)** 99

17. Each face of a cube is numbered with a positive integer in such a way that the numbers on pairs of faces sharing an edge differ by at least two. What is the minimum possible sum of six such integers?

(A) 12 **(B)** 15 **(C)** 18 **(D)** 24 **(E)** 27

18. The value of

$$1 + 2 + 3 + 4 - 5 + 6 + 7 + 8 + 9 - 10 + \cdots + 46 + 47 + 48 + 49 - 50$$

is equal to

(A) 600 **(B)** 650 **(C)** 725 **(D)** 750 **(E)** 800

Spring 2013 McNabb GDCTM Contest
Algebra One

NO Calculators Allowed

1. If 10 carpenters can build 10 cabinets in 10 days how many days does it take 20 carpenters to build 20 cabinets?

   (A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

2. The last 6 digits of 13^{426} are 000009. What is the sum of the last 6 digits of 13^{1704}?

   (A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

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4. If the integer $\underline{4400b074}$ is divisible by 101, what must the digit b equal?

   (A) 0 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 8

5. A rectangle with area 125 has its sides in the ratio of 4 : 5. What is the perimeter of this rectangle?

   (A) 18 **(B)** 22.5 **(C)** 36 **(D)** 45 **(E)** 54

6. For how many positive integers n does $n!$ end in exactly eleven zeros?

   (A) 0 **(B)** 3 **(C)** 5 **(D)** 8 **(E)** 11

7. Which of the following equations has exactly two solutions over the real numbers?

   (A) $x^2 - 6x + 9 = 0$ **(B)** $5x = 2(5 - 7x)$ **(C)** $|x + 8| = -5$
   (D) $|x| = 12$ **(E)** $x^2 + 1 = 0$

8. When a cyclist gets a puncture she has just completed three-fourths of her route. She finishes her route by walking. If she spent twice as much time walking as biking, how many times faster does she bike than walk?

   (A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

9. Given the three points $(2013, -1863)$, $(1776, -1812)$, and $(1181, -1492)$ in the coordinate plane, a fourth point (a, b) is called a *complementing* point if it along with the given three points form the vertices of a parallelogram. Find the sum of all the coordinates of all the complementing points of the given three points.

(A) -197 **(B)** 0 **(C)** 216 **(D)** 631 **(E)** 783

10. The sum of a set of numbers is the sum of all the numbers in that set. How many subsets of the set $\{1, 2, 3, 4, 5, 6, 7\}$ have a sum of 12?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

11. An amount of \$10000 dollars is deposited in an account for one year at an interest rate of x percent per year compounded twice a year. If, at the end of the year, \$10404 is in the account, then x is

(A) 3.9 **(B)** 4 **(C)** 4.1 **(D)** 7.8 **(E)** 8

12. Which transformation never changes the median of a list of a dozen distinct positive integers?

(A) adding 6 to each number in the list
(B) adding 3 to each of the three smallest numbers in the list
(C) subtracting 4 from each of the four largest numbers in the list
(D) doubling each number in the list
(E) taking the reciprocal of each number in the list

13. One factor of $14x^2 + 37x + 24$ is

(A) $2x + 1$ **(B)** $14x + 3$ **(C)** $2x + 7$ **(D)** $7x + 8$ **(E)** $7x + 3$

14. A purse may only contain pennies, nickels, dimes, and quarters but does not have to contain any particular type of coin, except as demanded in meeting the following conditions: the average value of the coins in the purse is 16 cents; if one more quarter were added to it the average value would rise to 17 cents. How many quarters are actually in the purse?

(A) 0 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** cannot be uniquely determined

15. For what value of the constant a do the three lines

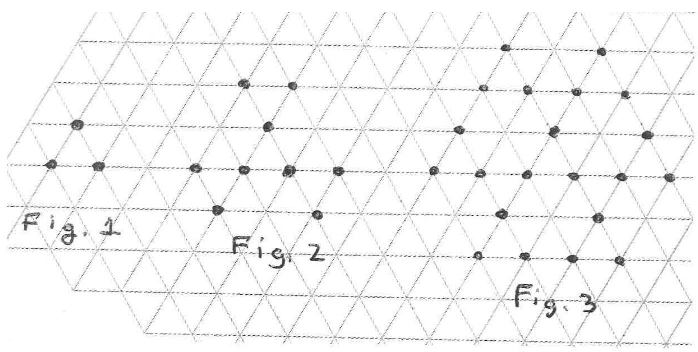
$$2x + 5y = -7 \qquad 3x - 2y = 18 \qquad ax + 6y = 2$$

all intersect at the same point?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

16. The first three figures of a certain sequence of figures are shown below on an equilateral triangle grid. Each successor figure is obtained recursively from its predeccesor by this rule: any two or more consecutive dots on a grid line generate new neighboring dots on that grid line, on either side, where no dot was before. All previous dots remain. How many dots does the 5th figure in this sequence have?

(A) 34 **(B)** 36 **(C)** 57 **(D)** 59 **(E)** 64



17. If the equations $x^2 + ax + 21 = 0$ and $2x^2 + 19x + 35 = 0$ have a solution in common, what could be the value of the constant a?

(A) -10 **(B)** -4 **(C)** -2 **(D)** 4 **(E)** 10

18. Amanda and Blake together can paint a house in 749 hours. Blake and Cathy together paint it in 535 hours. Cathy and Amanda together paint it in 642 hours. How long would it take in hours to paint the house if all three work together?

(A) 400 **(B)** 420 **(C)** 430 **(D)** 440 **(E)** 460

Spring 2013 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. Express the fraction

$$\frac{1}{1+\frac{1}{3+\frac{1}{5}}}$$

in lowest terms.

(A) 1/8 **(B)** 11/15 **(C)** 15/21 **(D)** 16/21 **(E)** 21/16

2. The square root of 20000 lies between

(A) 130 and 131 **(B)** 140 and 141 **(C)** 141 and 142
(D) 142 and 143 **(E)** 10,000 and 10,001

3. How many positive factors does 2013 have?

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4. Four indistinguishable roses and two indistinguishable tulips are to be arranged in a circle. Two such arrangements are considered to be the same if and only if each can be rotated into the other. How many distinct arrangements are possible?

(A) 3 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 15

5. When a cyclist gets a puncture she has just completed three-fourths of her route. She finishes her route by walking. If she spent twice as much time walking as biking, how many times faster does she bike than walk?

(A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

6. Which of the following equations has exactly two solutions over the real numbers?

(A) $x^2 - 6x + 9 = 0$ **(B)** $5x = 2(5 - 7x)$ **(C)** $|x + 8| = -5$
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(A) -10 **(B)** -4 **(C)** -2 **(D)** 4 **(E)** 10

8. An off-center balance does balance when pan A has a weight of 600 grams while pan B has a weight of 900 grams. If a weight of 400 grams is added to pan A, how many grams must be added to pan B to restore the balance? Neglect the mass of the pans, beams, etc...

(A) 400 **(B)** 500 **(C)** 600 **(D)** 700 **(E)** 900

9. Twenty-seven unit cubes are assembled to form a $3 \times 3 \times 3$ cube. If two of the unit cubes are then chosen at random, what is the probability they share a face?

(A) 2/13 **(B)** 3/11 **(C)** 1/4 **(D)** 3/16 **(E)** 1/3

10. In pentagon $ABCDE$, $AB = AE = 3$, $BC = DE = 1$, $CD = 3$, $\angle B = \angle E$, and $\angle A$ is right. The area of this pentagon lies between

(A) 6 and 7 **(B)** 7 and 8 **(C)** 8 and 9
(D) 9 and 10 **(E)** 10 and 11

11. A rectangle with area 125 has its sides in the ratio of 4 : 5. What is the perimeter of this rectangle?

(A) 18 **(B)** 22.5 **(C)** 36 **(D)** 45 **(E)** 54

12. A solid opaque cube of side length 5 meters rests on flat ground. It is illuminated only by a powerful point-source of light located 5 meters above one of the cube's top corners. Find the area in square meters of the shadow cast by the cube on the ground.

(A) 75 **(B)** 85 **(C)** $50\sqrt{2}$ **(D)** 91 **(E)** 100

13. For what value of the constant a do the three lines

$$2x + 5y = -7 \qquad 3x - 2y = 18 \qquad ax + 6y = 2$$

all intersect at the same point?

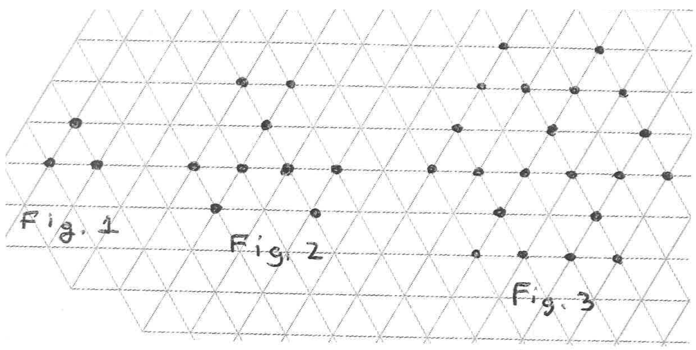
(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

14. Two sides of a parallelogram lie along the lines $x - y + 1 = 0$ and $2x + 3y - 6 = 0$. If the diagonals of the parallelogram meet at the point $(1, 1/2)$, find the area of this parallelogram.

(A) 3/2 **(B)** 2 **(C)** 5/2 **(D)** 18/7 **(E)** 3

15. The first three figures of a certain sequence of figures are shown below on an equilateral triangle grid. Each successor figure is obtained recursively from its predeccesor by this rule: any two or more consecutive dots on a grid line generate new neighboring dots on that grid line, on either side, where no dot was before. All previous dots remain. How many dots does the 5th figure in this sequence have?

(A) 34 **(B)** 36 **(C)** 57 **(D)** 59 **(E)** 64



16. Given the three points $(2013, -1863)$, $(1776, -1812)$, and $(1181, -1492)$ in the coordinate plane, a fourth point (a, b) is called a *complementing* point if it along with the given three points form the vertices of a parallelogram. Find the sum of all the coordinates of all the complementing points of the given three points.

(A) -197 **(B)** 0 **(C)** 216 **(D)** 631 **(E)** 783

17. Quadrilateral $PQRS$ is inscribed in a circle. Segments PQ and SR are extended to meet at T. If $\angle SPQ = 80°$ and $\angle PQR = 130°$, find in degrees the measure of $\angle T$.

(A) 50 **(B)** 53 **(C)** 57 **(D)** 60 **(E)** 61

18. A circle of radius 9 is externally tangent to a second circle of radius b. If a common tangent to the two circles has length 12, what is the value of b?

(A) 3.5 **(B)** 4 **(C)** 6 **(D)** 7.5 **(E)** 9

Spring 2013 McNabb GDCTM Contest
Algebra Two

NO Calculators Allowed

1. If 10% of a is b what is 10% of b?

   (A) $100a$ **(B)** $10a$ **(C)** a **(D)** $.1a$ **(E)** $.01a$

2. When a cyclist gets a puncture she has just completed three-fourths of her route. She finishes her route by walking. If she spent twice as much time walking as biking, how many times faster does she bike than walk?

   (A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

3. How many seconds are there in exactly six weeks?

   (A) 7! **(B)** 8! **(C)** 9! **(D)** 10! **(E)** 12!

4. If 10 carpenters can build 10 cabinets in 10 days how many days does it take 20 carpenters to build 20 cabinets?

   (A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

5. Which of the following equations has exactly two solutions over the real numbers?

   (A) $x^2 - 6x + 9 = 0$ **(B)** $5x = 2(5 - 7x)$ **(C)** $|x + 8| = -5$
   (D) $|x| = 12$ **(E)** $x^2 + 1 = 0$

6. In the following triangular arrangement of the positive integers, in which column, counting from left to right, does 7021 appear?

$$\begin{array}{cccc} & & & \cdot \\ & & 5 & \cdots \\ & 2 & 6 & \cdots \\ 1 & 3 & 7 & \cdots \\ & 4 & 8 & \cdots \\ & & 9 & \cdots \\ & & & \cdot \end{array}$$

   (A) 43 **(B)** 51 **(C)** 52 **(D)** 84 **(E)** 99

7. The number of digits in the large number 2^{50} is

(A) between 6 and 10 inclusive
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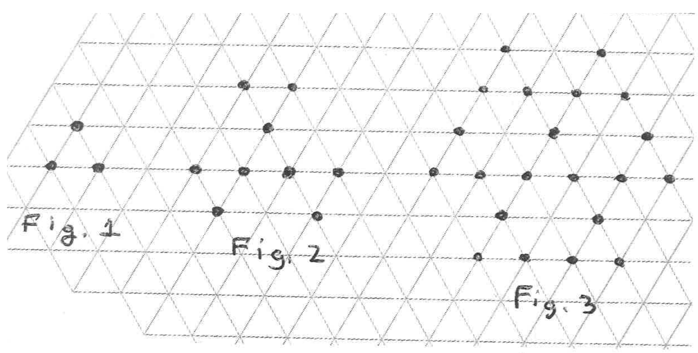

9. For how many positive integers n does $n!$ end in exactly eleven zeros?

(A) 0 **(B)** 3 **(C)** 5 **(D)** 8 **(E)** 11

10. Which of these numbers is the least?

(A) $\log_8 144$ **(B)** $\log_4 72$ **(C)** $\log_{16} 288$ **(D)** $\log_2 48$ **(E)** $\log_{32} 576$

11. Quadrilateral $PQRS$ is inscribed in a circle. Segments PQ and SR are extended to meet at T. If $\angle SPQ = 80°$ and $\angle PQR = 130°$, find in degrees the measure of $\angle T$.

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(A) -10 **(B)** -4 **(C)** -2 **(D)** 4 **(E)** 10

13. Which transformation never changes the median of a list of a dozen distinct positive integers?

(A) adding 6 to each number in the list
(B) adding 3 to each of the three smallest numbers in the list
(C) subtracting 4 from each of the four largest numbers in the list
(D) doubling each number in the list
(E) taking the reciprocal of each number in the list

14. How many different paths are there from $(0,0)$ to $(4,4)$ if only these three kinds of steps may be taken: (i) a unit step to the right, (ii) a unit step up, (iii) a northeast diagonal step from point (i,j) to point $(i+1, j+1)$?

(A) 276 **(B)** 295 **(C)** 321 **(D)** 343 **(E)** 371

15. How many solutions in radians of $\sin 2\theta = \cos 3\theta$ lie in the interval $[0, 2\pi]$?

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

16. Let $f(x) = (1/4)x^2 + bx + c$ where b and c are constants. If b and c are chosen randomly and independently from the set of digits $\{0,1,2,3,4,5,6,7,8,9\}$ what is the probability that the vertex of the parabola $y = f(x)$ lies on the x-axis?

(A) 1/25 **(B)** 1/20 **(C)** 1/10 **(D)** 4/25 **(E)** 1/5

17. Let a, b, and n be constants, with n a positive integer. If the first three terms of the binomial expansion of $(a+x)^n$ are, in ascending powers of x, equal to $3b + 6bx + 5bx^2$, then find the value of $a + b + n$.

(A) 48 **(B)** 64 **(C)** 96 **(D)** 128 **(E)** 252

18. When $x^{101} + x^{51} + 1$ is divided by $x^3 + 1$, what is the remainder?

(A) 0 **(B)** x **(C)** $3x^2 + 4x - 2$ **(D)** -1 **(E)** $-x^2$

SPRING 2013 MCNABB GDCTM CONTEST
PRECALCULUS

NO Calculators Allowed

1. A rectangle with area 125 has its sides in the ratio of $4 : 5$. What is the perimeter of this rectangle?

   (A) 18 **(B)** 22.5 **(C)** 36 **(D)** 45 **(E)** 54

2. In the repeating decimal $0.\overline{71771}$, in which decimal place does the 2013th 7 appear?

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4. The sum of a set of numbers is the sum of all the numbers in that set. How many subsets of the set $\{1,2,3,4,5,6,7\}$ have a sum of 12?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

5. The value of

   $$1+2+3+4-5+6+7+8+9-10+\cdots+46+47+48+49-50$$

   is equal to

   (A) 600 **(B)** 650 **(C)** 725 **(D)** 750 **(E)** 800

6. A solid opaque cube of side length 5 meters rests on flat ground. It is illuminated only by a powerful point-source of light located 5 meters above one of the cube's top corners. Find the area in square meters of the shadow cast by the cube on the ground.

   (A) 75 **(B)** 85 **(C)** $50\sqrt{2}$ **(D)** 91 **(E)** 100

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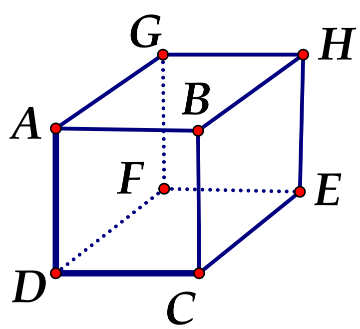
(A) 48 **(B)** 64 **(C)** 96 **(D)** 128 **(E)** 252

8. Which of these numbers is the least?

(A) $\log_8 144$ **(B)** $\log_4 72$ **(C)** $\log_{16} 288$ **(D)** $\log_2 48$ **(E)** $\log_{32} 576$

9. In cube $ABCDEFGH$ shown find $\cot \angle DBF$

(A) $2/\sqrt{6}$ **(B)** $5/6$ **(C)** 1 **(D)** $\sqrt{2}$ **(E)** $6/5$



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(D) $|x| = 12$ **(E)** $x^2 + 1 = 0$

11. How many solutions in radians of $\sin 2\theta = \cos 3\theta$ lie in the interval $[0, 2\pi]$?

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

12. Recall that $i = \sqrt{-1}$. What is the sum of the infinite geometric series $\sum_{n=0}^{\infty}(i/2)^n$?

(A) $-\frac{1}{5} + \frac{2}{5}i$ **(B)** $\frac{3}{5} - \frac{1}{5}i$ **(C)** $\frac{4}{5} + \frac{2}{5}i$ **(D)** 0 **(E)** i

13. Given the three points $(2013, -1863)$, $(1776, -1812)$, and $(1181, -1492)$ in the coordinate plane, a fourth point (a, b) is called a *complementing* point if it along with the given three points form the vertices of a parallelogram. Find the sum of all the coordinates of all the complementing points of the given three points.

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(A) $1/25$ **(B)** $1/20$ **(C)** $1/10$ **(D)** $4/25$ **(E)** $1/5$

16. A careless librarian has reshelved the 5 volumes of an art encyclopedia in the correct order. Each volume has its spine facing out, which is correct of course, but has a $1/4$ probability of being upside down. What is the probability that exactly one pair of front covers are now face to face?

(A) $1/64$ **(B)** $2/31$ **(C)** $3/16$ **(D)** $5/24$ **(E)** $69/128$

17. The set of points in space equidistant from two skew lines is

(A) the empty set **(B)** a single point **(C)** a line
(D) the union of two intersecting lines **(E)** none of the above

18. In triangle ABC, the angle bisector CD of $\angle C$ has point D on side AB. If $AC = 1$, $BC = \sqrt{3}$, $AD = \sqrt{3} - 1$ and $DB = 3 - \sqrt{3}$, then what is the length CD?

(A) $\sqrt{1 + \sqrt{3}}$ **(B)** $\sqrt{6 - 3\sqrt{3}}$ **(C)** $9/10$ **(D)** 1 **(E)** $1/\sqrt{2}$

Spring 2013 McNabb GDCTM Contest
Calculus

NO Calculators Allowed

Assume all variables are real unless otherwise stated in the problem.

1. How many positive factors does 2013 have?

   (A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

2. The value of

   $$1+2+3+4-5+6+7+8+9-10+\cdots+46+47+48+49-50$$

   is equal to

   (A) 600 **(B)** 650 **(C)** 725 **(D)** 750 **(E)** 800

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(A) 1/64 **(B)** 2/31 **(C)** 3/16 **(D)** 5/24 **(E)** 69/128

8. Recall that $i = \sqrt{-1}$. What is the sum of the infinite geometric series $\sum_{n=0}^{\infty}(i/2)^n$?

(A) $-\frac{1}{5}+\frac{2}{5}i$ **(B)** $\frac{3}{5}-\frac{1}{5}i$ **(C)** $\frac{4}{5}+\frac{2}{5}i$ **(D)** 0 **(E)** i

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(A) the empty set **(B)** a single point **(C)** a line
(D) the union of two intersecting lines **(E)** none of the above

10. How many solutions in radians of $\sin 2\theta = \cos 3\theta$ lie in the interval $[0, 2\pi]$?

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

11. The integral

$$\int_0^{\pi/2} \frac{1}{1+\cos\theta}\, d\theta$$

has value

(A) 3/5 **(B)** 5/6 **(C)** 1 **(D)** 7/5 **(E)** diverges

12. Find the minimum possible value of the expression $6\cos x + 2\cos 2x + 5$.

(A) 2/3 **(B)** 3/4 **(C)** 4/5 **(D)** 5/6 **(E)** 1

13. A thin rod lies along the x-axis with endpoints at $x = 2$ and $x = 8$. If the density of the rod at each point is directly proportional to the point's distance to the origin, what is the x-coordinate of the center of mass of the rod?

(A) 19/5 **(B)** 4 **(C)** 14/3 **(D)** 28/5 **(E)** 5

14. How many values of the constant k satisfy both: (i) $k \geq 1$ and (ii) $\int_1^k (2k-2)x^k \, dx = 80$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

15. Determine

$$\lim_{n\to\infty} \int_0^{\pi/6} (\sin x)^n \, dx$$

(A) 0 **(B)** 1/10 **(C)** $\pi/12$ **(D)** 1/2 **(E)** does not exist

16. The improper integral $\int_0^\infty \frac{1}{1+e^x} \, dx$ has the value

(A) $\ln 2$ **(B)** 1/2 **(C)** 2/3 **(D)** e **(E)** does not converge

17. Given that $\int_0^{10} \ln(x^2 - 10x + 26) \, dx = k$ then find the value of

$$\int_0^{10} x \ln(x^2 - 10x + 26) \, dx$$

(A) 0 **(B)** k **(C)** $2k$ **(D)** $k \ln 2$ **(E)** $5k$

18. The coefficient of x^8 in the Maclaurin power series of $f(x) = \dfrac{1+2x}{1-x-x^2}$ is equal to

(A) 47 **(B)** 76 **(C)** 91 **(D)** 101 **(E)** 123