Spring 2012 McNabb GDCTM Contest
Prealgebra

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. How many of the first 20 natural numbers, that is, the set $\{1, 2, 3, \cdots, 20\}$, are composite? Recall that 1 is neither prime nor composite.

   (A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

2. The value of $-3 - 3^2 - 3^3$ is equal to

   (A) -41 **(B)** -40 **(C)** -39 **(D)** -37 **(E)** -36

3. A clever saleswoman is counting out envelopes for a customer. Every package of envelopes contains 80 envelopes. The saleswoman can count out 8 envelopes in 8 seconds. How many seconds does she need to count out 56 envelopes?

   (A) 24 **(B)** 48 **(C)** 56 **(D)** 72 **(E)** 80

4. A jar contains only blue and green marbles in the ratio of 3 blue for every 5 green. If 3 green marbles are removed and replaced by 3 blue marbles, the ratio of blue to green marbles becomes one to one. How many green marbles were in the jar originally?

   (A) 15 **(B)** 18 **(C)** 21 **(D)** 24 **(E)** 27

5. What is the largest possible product of two positive odd integers whose sum is 40?

   (A) 39 **(B)** 279 **(C)** 300 **(D)** 399 **(E)** 400

6. What is the remainder when the sum

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3$$

is divided by 9?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

7. Before a grape is dried to become a raisin it is 94% water, while a raisin is only 25% water. If only water is evaporated from the grapes, how many kilograms of raisins can be made from 60 kilograms of grapes?

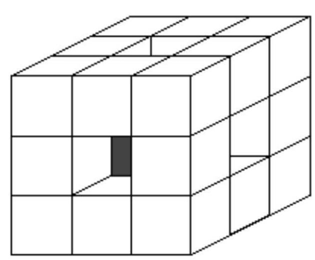
(A) 4.8 **(B)** 18.2 **(C)** 24.6 **(D)** 27.6 **(E)** 45

8. Mr. and Mrs. Reynolds have three daughters and three sons. At Easter each member of the family buys one chocolate Easter egg for everybody else in the family. How many Easter eggs will the Reynolds family buy in total?

(A) 28 **(B)** 32 **(C)** 40 **(D)** 56 **(E)** 64

9. Twenty seven small $1 \times 1 \times 1$ cubes are glued together to form a $3 \times 3 \times 3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

(A) 56 **(B)** 64 **(C)** 72 **(D)** 84 **(E)** 96



10. A train 2700 meters long passes a signal in 135 seconds. At that rate, how many seconds does it take to cross a bridge 1000 meters in length?

(A) 50 **(B)** 110 **(C)** 135
(D) 160 **(E)** 185

11. Which integer below cannot be written as the sum of the squares of two integers?

(A) 289 **(B)** 353 **(C)** 450 **(D)** 481 **(E)** 503

12. The digits 7,5,3,2, and 0 are each used once to make the smallest possible 5 digit number divisible by 11. What is the hundred's digit of this number?

(A) 0 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 7

13. A cylindrical can of juice holds 12 ounces. If the diameter of the can were to be doubled and the height halved, how many ounces would the new can hold?

(A) 6 **(B)** 12 **(C)** 24 **(D)** 48 **(E)** 96

14. If the area of a circle is increased by 300% by what percent is the circumference increased?

(A) 50% **(B)** 100% **(C)** 150% **(D)** 200% **(E)** 300%

15. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

(A) 104 **(B)** 108 **(C)** 148 **(D)** 158 **(E)** 266

16. A cage contains birds and rabbits. There are seventeen heads and forty feet. How many rabbits are in the cage?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 15

17. Water flows continuously into a 200ℓ tank at the rate of $3\ell/\text{min}$ and flows continuously out at the rate of $2\ell/\text{min}$. If the tank is initially 25% full, how many minutes will it take for the tank to fill completely?

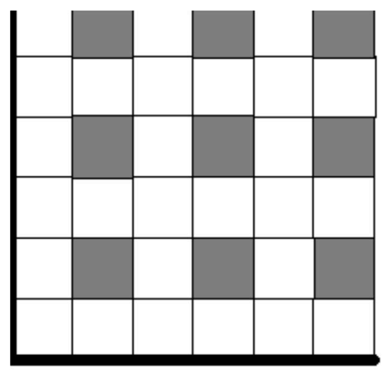
(A) 100 **(B)** 150 **(C)** 175 **(D)** 200 **(E)** 225

18. Jane took 5 tests, each time receiving a different score. These scores were all integers less than or equal to 100 and greater than or equal to zero. The average of her three lowest scoring tests was 84 while the average of her three highest scoring tests was 89. What is the maximum possible score of her highest scoring test?

(A) 95 **(B)** 96 **(C)** 97 **(D)** 98 **(E)** 99

19. A rectangular lobby is to be tiled in this pattern in such a way that the border tiles along all the edges of the lobby are white. If the lobby measures 201 tiles by 101 tiles, how many shaded tiles are required?

(A) 4900 **(B)** 4950 **(C)** 5000 **(D)** 5050 **(E)** 5100



20. The value of $\left(\sqrt{12}+\sqrt{3}\right)^2$ is

(A) 40 **(B)** 36 **(C)** 27 **(D)** 21 **(E)** 15

21. Quadrilateral $ABCD$ has vertices in the coordinate plane as follows: $A = (0,0), B = (5,12), C = (-3,-3)$, and $D = (0,-7)$. The perimeter of this polygon equals

(A) 28 **(B)** 35 **(C)** 42 **(D)** 45 **(E)** 46

22. Let

$$S = \frac{1+2+4+8+16}{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}}$$

Then S equals

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

23. Three pairs of husbands and wives are to be seated at a bolted-down picnic table which seats exactly six people, three to a side. If no husband is to sit on the same side as his wife, and no wife is to sit directly across from her husband, in how many ways can these six persons be seated?

(A) 24 **(B)** 60 **(C)** 64 **(D)** 80 **(E)** 96

24. For how many positive integers n is the least common multiple of 30, 24, and n equal to 600?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 18

25. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

(A) 5/12 **(B)** 1/11 **(C)** 1/24 **(D)** 1/22 **(E)** 1/48

Spring 2012 McNabb GDCTM Contest
Algebra One

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. The value of $-3 - 3^2 - 3^3$ is equal to

   (A) -40 **(B)** -39 **(C)** -37 **(D)** -36 **(E)** -33

2. A clever saleswoman is counting out envelopes for a customer. Every package of envelopes contains 80 envelopes. The saleswoman can count out 8 envelopes in 8 seconds. How many seconds does she need to count out 56 envelopes?

   (A) 24 **(B)** 48 **(C)** 56 **(D)** 72 **(E)** 80

3. If $a \diamond b$ equals the lesser of $1/a$ and $1/b$, find the value of $-3 \diamond (-2 \diamond (-1/2))$.

   (A) -3 **(B)** -1/2 **(C)** -1/3 **(D)** -2 **(E)** -1

4. What is the largest possible product of two positive odd integers whose sum is 40?

   (A) 39 **(B)** 279 **(C)** 300 **(D)** 399 **(E)** 400

5. Let
$$S = \frac{1+2+4+8+16}{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}}$$
   Then S equals

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

6. A cage contains birds and rabbits. There are seventeen heads and forty feet. How many rabbits are in the cage?

   (A) 0 **(B)** 3 **(C)** 6 **(D)** 9 **(E)** 12

7. Mr. and Mrs. Reynolds have three daughters and three sons. At Easter each member of the family buys one chocolate Easter egg for everyone else in the family. How many Easter eggs will the Reynolds family buy in total?

   (A) 28   **(B)** 32   **(C)** 40   **(D)** 56   **(E)** 64

8. Three pairs of husbands and wives are to be seated at a bolted-down picnic table which seats exactly six people, three to a side. If no husband is to sit on the same side as his wife, and no wife is to sit directly across from her husband, in how many ways can these six persons be seated?

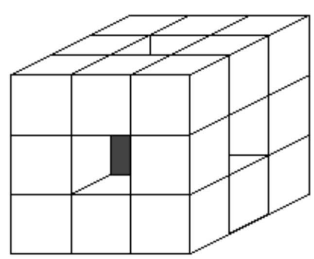
   (A) 24   **(B)** 60   **(C)** 64   **(D)** 80   **(E)** 96

9. Which integer below cannot be written as the sum of the squares of two integers?

   (A) 289   **(B)** 353   **(C)** 450   **(D)** 481   **(E)** 503

10. Twenty seven small $1 \times 1 \times 1$ cubes are glued together to form a $3 \times 3 \times 3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

   (A) 56   **(B)** 64   **(C)** 72   **(D)** 84   **(E)** 96



11. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

   (A) 104   **(B)** 108   **(C)** 148   **(D)** 158   **(E)** 266

12. The value of $\left(\sqrt{12}+\sqrt{3}\right)^2$ is

(A) 40 **(B)** 36 **(C)** 30 **(D)** 27 **(E)** 24

13. Quadrilateral $ABCD$ has vertices in the coordinate plane as follows: $A = (0,0), B = (5,12), C = (-3,-3)$, and $D = (0,-7)$. The perimeter of this polygon equals

(A) 28 **(B)** 35 **(C)** 42 **(D)** 45 **(E)** 46

14. What is the remainder when the sum

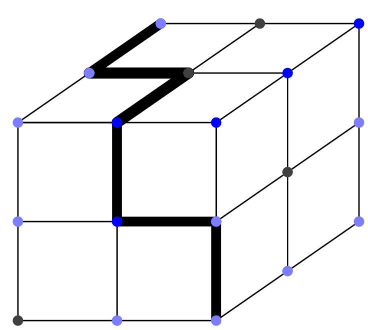
$$1^{111} + 2^{111} + 3^{111} + 4^{111} + 5^{111} + 6^{111} + 7^{111} + 8^{111} + 9^{111} + 10^{111}$$

is divided by 11?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

15. A small bug crawls on the surface of a $2 \times 2 \times 2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1 \times 1 \times 1$ cubes. How many shortest paths of this type are possible? One example is shown.

(A) 54 **(B)** 64 **(C)** 90 **(D)** 96 **(E)** 120



16. Given that $x \neq 0$, $y \neq 0$, and

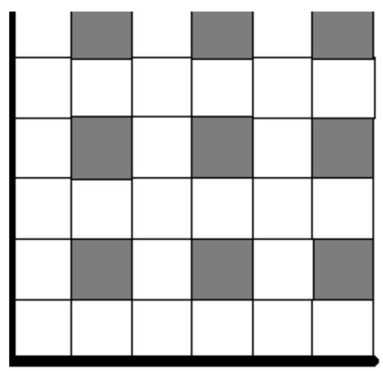
$$x = \frac{6}{y} - \frac{9}{xy^2}$$

what is the value of xy?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** cannot be uniquely determined

17. A rectangular lobby is to be tiled in this pattern in such a way that the border tiles along all the edges of the lobby are white. If the lobby measures 201 tiles by 101 tiles, how many shaded tiles are required?

(A) 4900 **(B)** 4950 **(C)** 5000 **(D)** 5050 **(E)** 5100

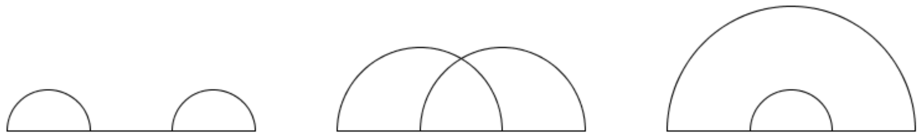


18. Jane took 5 tests, each time receiving a different score. These scores were all integers less than or equal to 100 and greater than or equal to zero. The average of her three lowest scoring tests was 84 while the average of her three highest scoring tests was 89. What is the maximum possible score of her highest scoring test?

(A) 94 **(B)** 95 **(C)** 96 **(D)** 97 **(E)** 98

19. In rectangle $ABCD$ point P is located on side CD, closer to C than D, in such a way that $\angle APB$ is right. If $AB = 5$ and $AD = 2$, find the length of segment CP.

(A) $5/7$ **(B)** $6/7$ **(C)** 1 **(D)** 2 **(E)** 3



20. Distribute 14 points along a line segment. How many distinct ways are there for pairing these points using semicircles? The case of four points is pictured above.

    (A) 10395 **(B)** 40320 **(C)** 60125 **(D)** 101245 **(E)** 135135

21. Three dice are rolled and it is known that the sum is a multiple of three. What is the probability that the sum is nine?

    (A) 1/3 **(B)** 25/72 **(C)** 13/48 **(D)** 1/2 **(E)** 11/18

22. For how many integer values of k can the polynomial $12x^2 + kx + 12$ be factored as $(ax+b)(cx+d)$ where a, b, c, and d are integers with $a \neq 0$ and $b \neq 0$?

    (A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

23. Find the coefficient of x in the expansion of

$$(x-2012)(x-2011)(x-2010)\cdots(x+2010)(x+2011)(x+2012)$$

    (A) $-(2012!)^2$ **(B)** $-(1006!)^2$ **(C)** 0 **(D)** $(1006!)^2$ **(E)** $(2012!)^2$

24. If $16^N = 2^1 \cdot 2^3 \cdot 2^6 \cdot 2^{10} \cdots 2^k$, where the exponents follow the triangular numbers, and k is the 20th triangular number, what is the value of N?

    (A) 190 **(B)** 215 **(C)** 300 **(D)** 385 **(E)** 420

25. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

    (A) 5/12 **(B)** 1/11 **(C)** 1/24 **(D)** 1/22 **(E)** 1/48

Spring 2012 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. A cage contains birds and rabbits. There are seventeen heads and forty feet. How many rabbits are in the cage?

   (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

2. Mr. and Mrs. Reynolds have three daughters and three sons. At Easter each member of the family buys one chocolate Easter egg for everyone else in the family. How many Easter eggs will the Reynolds family buy in total?

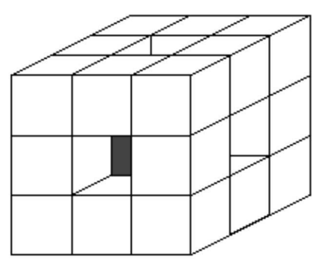
   (A) 28 (B) 32 (C) 40 (D) 56 (E) 64

3. Quadrilateral $ABCD$ has vertices in the coordinate plane as follows:
   $A = (0, 0), B = (5, 12), C = (-3, -3)$, and $D = (0, -7)$. The perimeter of this polygon equals

   (A) 40 (B) 42 (C) 45 (D) 46 (E) 48

4. Twenty seven small $1 \times 1 \times 1$ cubes are glued together to form a $3 \times 3 \times 3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

   (A) 56 (B) 64 (C) 72 (D) 84 (E) 96



5. What is the y-intercept of the perpendicular bisector of the segment with endpoints $(-2, 8)$ and $(8, 4)$?

   (A) -1/2 (B) 1/2 (C) 0 (D) -3/2 (E) 3/2

6. Three pairs of husbands and wives are to be seated at a bolted-down picnic table which seats exactly six people, three to a side. If no husband is to sit on the same side as his wife, and no wife is to sit directly across from her husband, in how many ways can these six persons be seated?

   (A) 24 (B) 60 (C) 64 (D) 80 (E) 96

7. What is the area of a triangle whose sides measure 13, 14, and 15?

   (A) 64 (B) 84 (C) 90 (D) 92 (E) 96

8. In rectangle $ABCD$ point P is located on side CD, closer to C than D, in such a way that $\angle APB$ is right. If $AB = 5$ and $AD = 2$, find the length of segment CP.

   (A) 1 (B) 6/5 (C) 5/4 (D) $\sqrt{2}$ (E) 2

9. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

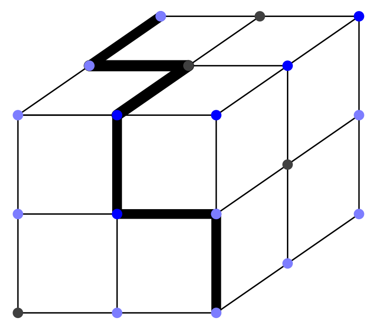
   (A) 104 (B) 108 (C) 148 (D) 158 (E) 266

10. In $\triangle ABC$ points D, E, and F lie on segments $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ respectively, in such a way that the proportions $BD/DC = 7/3$, $CE/EA = 3/2$, and $AF/FB = 4/1$ hold. If AD and FE intersect at G, what is the ratio AG/GD?

    (A) 5/6 (B) 6/7 (C) 7/8 (D) 8/9 (E) 1/1

11. A small bug crawls on the surface of a $2 \times 2 \times 2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1 \times 1 \times 1$ cubes. How many shortest paths of this type are possible? One example is shown.

(A) 54 (B) 64 (C) 90 (D) 96 (E) 120



12. Hezy and Zeke are both painters. Working alone Hezy can paint a certain room in 4 hours. Working alone Zeke can paint the same room in 6 hours. Let r_H, r_Z, and r_{HZ} be respectively the rates at which Hezy works alone, Zeke works alone, and Hezy and Zeke work together. Suppose for some coefficient of efficiency k satisfying $0 \leq k \leq 1$, these rates are related by the formula

$$r_{HZ} = k(r_H + r_Z)$$

Very often $k < 1$ since the two working at the same time interfere with each other to some extent. What is the value of k if in fact it takes Hezy and Zeke working together 2.7 hours to paint this room?

(A) 6/7 (B) 7/8 (C) 8/9 (D) 9/10 (E) 10/11

13. In $\triangle ABC$, the medians AD and BE are perpendicular. If $AC = 8$ and $BC = 12$, what is the length of AB?

(A) 6 (B) $4\sqrt{13/5}$ (C) 9 (D) $4\sqrt{3}$ (E) 7

14. Consider the lines $y = 0, y = \sqrt{3}$, and $y = \sqrt{3}x$. Let C be the center of the circle that is both tangent to all three of these lines and whose x-coordinate is negative. The sum of the coordinates for the center of C can written in the form $a + b\sqrt{3}$ where a and b are rational numbers. Determine $a + b$.

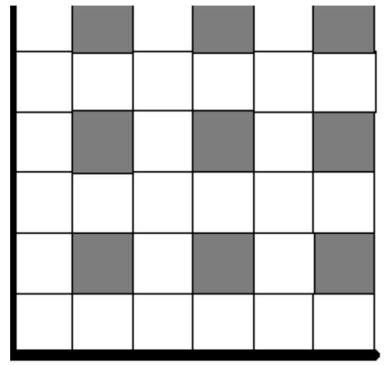
(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

15. A rectangular lobby is to be tiled in this pattern in such a way that the border tiles along all the edges of the lobby are white. If the lobby measures 201 tiles by 101 tiles, how many shaded tiles are required?

(A) 4900 (B) 4950 (C) 5000 (D) 5050 (E) 5100

16. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

(A) 5/12 (B) 1/11 (C) 1/24 (D) 1/22 (E) 1/48

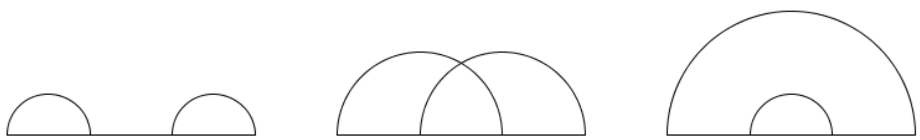


17. Let points A, B, C, and P lie in the $x-y$ plane. The coordinates of A, B, and C are $(0,0)$, $(3,0)$, and $(9,0)$ respectively. If $AP = 7$ and $BP = 6$, what is the value of CP?

    (A) 8 (B) $\sqrt{65}$ (C) 25/3 (D) 9 (E) $\sqrt{85}$

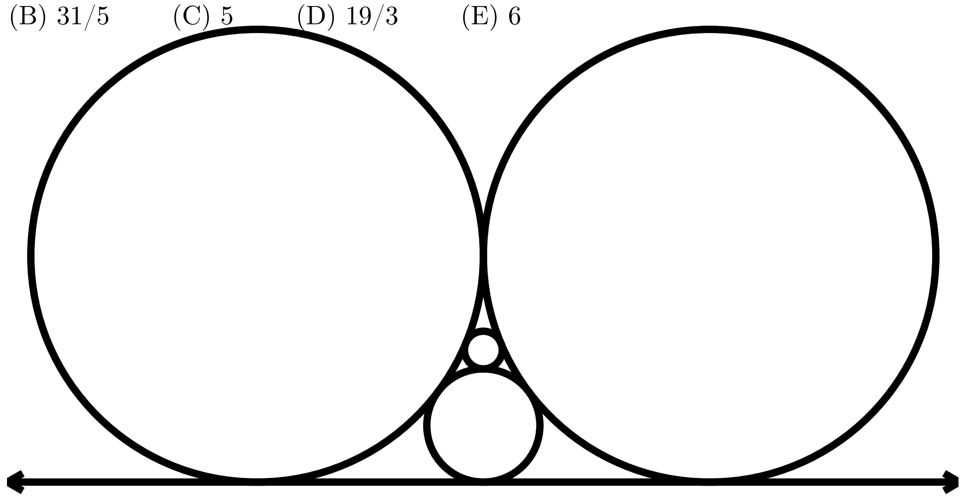
18. Distribute 14 points along a line segment. How many distinct ways are there for pairing these points using semicircles? The case of four points is pictured below.

    (A) 10395 (B) 40320 (C) 60125 (D) 101245 (E) 135135



19. Two congruent large circles and a smaller third circle are mutually externally tangent and are also tangent to the same line, as shown. A fourth circle of diameter one, smaller than the rest, is drawn tangent to these three circles. What is the radius of the two large congruent circles?

    (A) $4\sqrt{2}$ (B) 31/5 (C) 5 (D) 19/3 (E) 6



20. Let $\angle ABC$ measure 30 degrees. Imagine the rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are silvered as a mirror to reflect light. For a light beam that starts anywhere in the interior of $\angle ABC$, what is the maximum number of times such a beam can strike these mirrors?

    (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Spring 2012 McNabb GDCTM Contest
Algebra II

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. The value of $\left(\sqrt{12}+\sqrt{3}\right)^2$ is

   (A) 36 **(B)** 27 **(C)** 21 **(D)** 15 **(E)** 12

2. A cage contains birds and rabbits. There are seventeen heads and forty feet. How many rabbits are in the cage?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

3. Mr. and Mrs. Reynolds have three daughters and three sons. At Easter each member of the family buys one chocolate Easter egg for everyone else in the family. How many Easter eggs will the Reynolds family buy in total?

   (A) 28 **(B)** 32 **(C)** 40 **(D)** 56 **(E)** 64

4. If $a \diamond b$ equals the lesser of $1/a$ and $1/b$, find the value of $-3 \diamond (-2 \diamond (-1/2))$.

   (A) -3 **(B)** -1/3 **(C)** -1/2 **(D)** -2 **(E)** -1

5. What is the y-intercept of the perpendicular bisector of the segment with endpoints $(-2, 8)$ and $(8, 4)$?

   (A) -1/2 **(B)** 1/2 **(C)** 0 **(D)** -3/2 **(E)** 3/2

6. The graph of $z^2 = 4x^2 + 4y^2$ is a double cone and the graph of $2x - z = 8$ is a plane. The intersection of these two graphs is

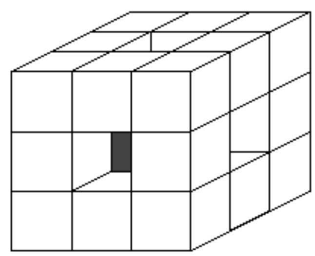
   (A) a circle **(B)** a non-circular ellipse **(C)** an hyperbola
   (D) a parabola **(E)** two intersecting lines

7. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

(A) 104 **(B)** 108 **(C)** 148 **(D)** 158 **(E)** 266

8. Twenty seven small $1 \times 1 \times 1$ cubes are glued together to form a $3 \times 3 \times 3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

(A) 56 **(B)** 64 **(C)** 72 **(D)** 84 **(E)** 96



9. If $2 + \ln x = \ln(x+2)$ then x must equal

(A) $\dfrac{2}{e^2-1}$ **(B)** $\dfrac{2}{e-1}$ **(C)** $\dfrac{1}{e^2-1}$ **(D)** $\dfrac{2}{e^2+1}$ **(E)** $\dfrac{1}{e^2+1}$

10. Hezy and Zeke are both painters. Working alone Hezy can paint a certain room in 4 hours. Working alone Zeke can paint the same room in 6 hours. Let r_H, r_Z, and r_{HZ} be respectively the rates at which Hezy works alone, Zeke works alone, and Hezy and Zeke work together. Suppose for some coefficient of efficiency k satisfying $0 \leq k \leq 1$, these rates are related by the formula

$$r_{HZ} = k(r_H + r_Z)$$

Very often $k < 1$ since the two working at the same time interfere with each other to some extent. What is the value of k if in fact it takes Hezy and Zeke working together 2.7 hours to paint this room?

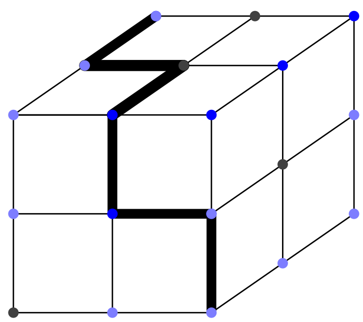
(A) 6/7 **(B)** 7/8 **(C)** 8/9 **(D)** 9/10 **(E)** 10/11

11. If $a^2 - 2a + b^2 - 2b = ab - 4$, then what is the value of $a + 2b$?

(A) 0 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** cannot be uniquely determined

12. A small bug crawls on the surface of a $2 \times 2 \times 2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1 \times 1 \times 1$ cubes. How many shortest paths of this type are possible? One example is shown.

(A) 54 **(B)** 64 **(C)** 90 **(D)** 96 **(E)** 120



13. Find the value of $r^2s^2 + s^2t^2 + t^2r^2$ if r, s, and t are the three possibly complex roots of the cubic polynomial $x^3 + 5x^2 - 3x + 1$.

(A) -1 **(B)** 0 **(C)** 3 **(D)** 5 **(E)** 8

14. In $\triangle ABC$ points D, E, and F lie on segments $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ respectively, in such a way that the proportions $BD/DC = 7/3$, $CE/EA = 3/2$, and $AF/FB = 4/1$ hold. If AD and FE intersect at G, what is the ratio AG/GD?

(A) 5/6 **(B)** 6/7 **(C)** 7/8 **(D)** 8/9 **(E)** 1/1

15. If $x > \frac{1}{x}$, then which of the following must be true?

I. $2x > \frac{2}{x}$

II. $2x > \frac{1}{2x}$

III. $x^2 > \frac{1}{x^2}$

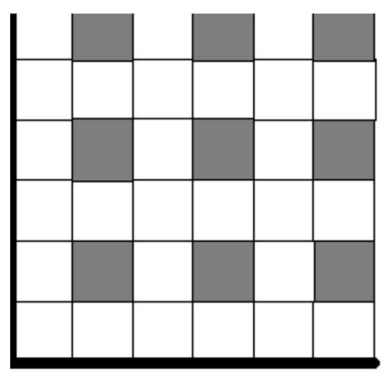
(A) I only **(B)** I and II only **(C)** I and III only **(D)** II and III only
(E) I, II, and III

16. When the polynomial x^{2012} is divided by the polynomial $x^2 + x + 1$ what is the remainder $R(x)$?

(A) -1 **(B)** $x + 1$ **(C)** $2x - 1$ **(D)** 0 **(E)** $-x - 1$

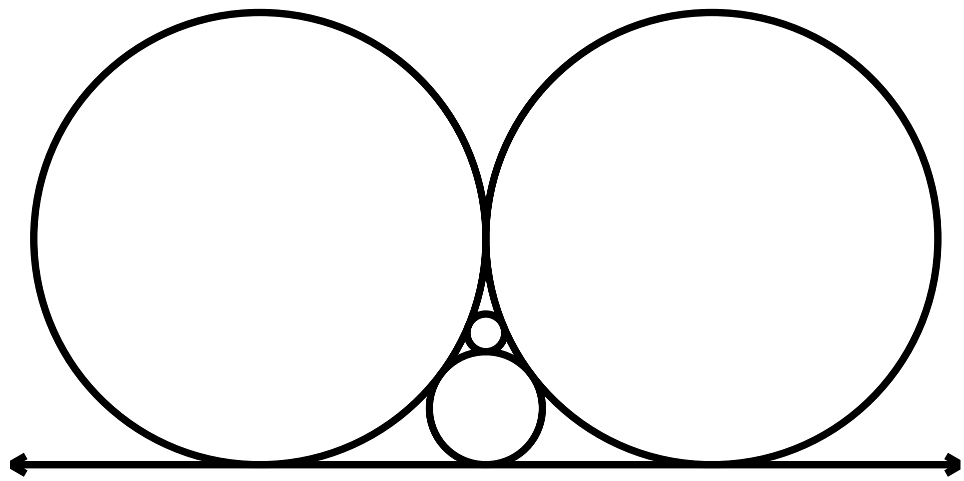
17. A rectangular lobby is to be tiled in this pattern in such a way that the border tiles along all the edges of the lobby are white. If the lobby measures 201 tiles by 101 tiles, how many shaded tiles are required?

(A) 4900 **(B)** 4950 **(C)** 5000 **(D)** 5050 **(E)** 5100



18. Two congruent large circles and a smaller third circle are mutually externally tangent and also tangent to the same line, as shown. A fourth circle of diameter one, smaller than the rest, is drawn tangent to these three circles. What is the radius of the two large congruent circles?

(A) $4\sqrt{2}$ **(B)** $31/5$ **(C)** 5 **(D)** $19/3$ **(E)** 6



19. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

(A) $5/12$ **(B)** $1/11$ **(C)** $1/24$ **(D)** $1/22$ **(E)** $1/48$

20. Let $\angle ABC$ measure 30 degrees. Imagine the rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are silvered as a mirror to reflect light. For a light beam that starts anywhere in the interior of $\angle ABC$, what is the maximum number of times such a beam can strike these mirrors?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

Spring 2012 McNabb GDCTM Contest
Pre-calculus

NO Calculators Allowed

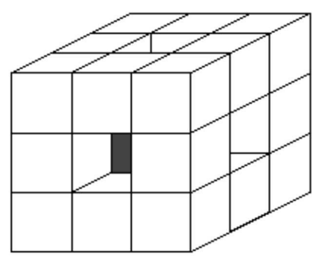
Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. The value of $-3-3^2-3^3$ is equal to

   (A) -41 **(B)** -40 **(C)** -39 **(D)** -37 **(E)** -36

2. Twenty seven small $1\times1\times1$ cubes are glued together to form a $3\times3\times3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

   (A) 56 **(B)** 64 **(C)** 72 **(D)** 84 **(E)** 96



3. What is the remainder when the sum

$$1^{111}+2^{111}+3^{111}+4^{111}+5^{111}+6^{111}+7^{111}+8^{111}+9^{111}+10^{111}$$

   is divided by 11?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

4. In triangle ABC, put $AB=c$, $BC=a$, and $CA=b$. If $(a+b+c)(a+b-c)=ab$, what is the degree measure of $\angle C$?

   (A) 120 **(B)** 105 **(C)** 90 **(D)** 75 **(E)** 60

5. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

(A) 104 **(B)** 108 **(C)** 148 **(D)** 158 **(E)** 266

6. When expanded and simplified $(1 - x + x^2 - x^3)^{10}$ has the form

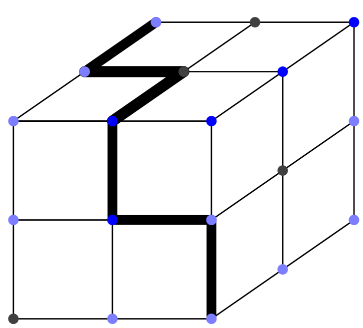
$$c_0 + c_1 x + c_2 x^2 + \cdots + c_{30} x^{30}$$

What is the value of $c_1 + c_3 + c_5 + \cdots + c_{29}$, the sum of the coefficients of all the odd powers of x?

(A) -4^{19} **(B)** -2^{20} **(C)** -2^{19} **(D)** 0 **(E)** 2^{19}

7. A small bug crawls on the surface of a $2 \times 2 \times 2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1 \times 1 \times 1$ cubes. How many shortest paths of this type are possible? One example is shown.

(A) 54 **(B)** 64 **(C)** 90 **(D)** 96 **(E)** 120



8. If $\sin x + \cos x = \sqrt{5}/3$, then what is the value of $\sqrt{\cos 4x}$?

(A) $5/9$ **(B)** $7/9$ **(C)** $7/11$ **(D)** $8/11$ **(E)** $12/13$

9. Find the value of $r^2s^2 + s^2t^2 + t^2r^2$ if r, s, and t are the three possibly complex roots of the cubic polynomial $x^3 + 5x^2 - 3x + 1$.

(A) -1 **(B)** 0 **(C)** 3 **(D)** 5 **(E)** 8

10. What is the sum of all the odd 5 digit numbers in which each of the digits 1,2,3,4, and 5 occur exactly once?

(A) 2000976 **(B)** 2188876 **(C)** 2299936 **(D)** 2399976 **(E)** 2499936

11. If $a^2 - 2a + b^2 - 2b = ab - 4$, then what is the value of $a + 2b$?

(A) 0 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** cannot be uniquely determined

12. Consider the lines $y = 0$, $y = \sqrt{3}$, and $y = x\sqrt{3}$. Let C be the center of the circle tangent to all three lines such that the x-coordinate of this center is negative. The sum of the coordinates of C can be written in the form $a + b\sqrt{3}$ where a and b are rational numbers. Find $a + b$.

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

13. The graph of $z^2 = 4x^2 + 4y^2$ is a double cone and the graph of $2x - z = 8$ is a plane. The intersection of these two graphs is

(A) a circle **(B)** a non-circular ellipse **(C)** an hyperbola
(D) a parabola **(E)** two intersecting lines

14. What is the sum

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{8} + \frac{1}{10} + \frac{1}{16} + \frac{1}{20} + \frac{1}{25} + \cdots$$

which consists of the sum of the reciprocals of 1 and all the natural numbers whose prime factorizations contain no primes other than 2 or 5?

(A) 5/2 **(B)** 11/4 **(C)** 19/8 **(D)** 17/8 **(E)** 3

15. If $2 + \ln x = \ln(x+2)$ then x must equal

(A) $\dfrac{2}{e^2-1}$ **(B)** $\dfrac{2}{e-1}$ **(C)** $\dfrac{1}{e^2-1}$ **(D)** $\dfrac{2}{e^2+1}$ **(E)** $\dfrac{1}{e^2+1}$

16. In $\triangle ABC$, the medians AD and BE are perpendicular. If $AC = 8$ and $BC = 12$, what is the length of AB?

(A) 6 **(B)** $4\sqrt{13/5}$ **(C)** $4\sqrt{3}$ **(D)** 7 **(E)** 9

17. When the polynomial x^{2012} is divided by the polynomial $x^2 + x + 1$ what is the remainder $R(x)$?

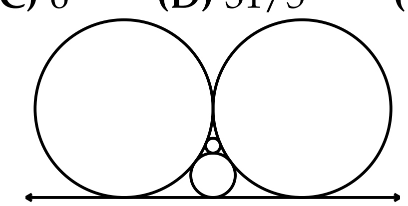
(A) 1 **(B)** $-x-1$ **(C)** $x+1$ **(D)** $2x-1$ **(E)** 0

18. A 2012×2012 matrix A has its entry in the ith row and jth column designated by $a_{i,j}$. Suppose for each $k = 1, 2, 3, \cdots, 1006$ that $a_{2k-1,2k-1} = 4k-3$, $a_{2k-1,2k} = 4k-2$, $a_{2k,2k-1} = 4k-1$, and $a_{2k,2k} = 4k$. All other values of $a_{i,j}$ are set equal to zero. Find the value of $\det A$, where $\det A$ stands for the determinant of A.

(A) -4^{1006} **(B)** -2^{1006} **(C)** 0 **(D)** 2^{1006} **(E)** 4^{1006}

19. Two congruent large circles and a smaller third circle are mutually externally tangent and also tangent to the same line, as shown. A fourth circle of diameter one, smaller than the rest, is drawn tangent to these three circles. What is the radius of the two large congruent circles?

(A) 5 **(B)** $4\sqrt{2}$ **(C)** 6 **(D)** $31/5$ **(E)** $19/3$



20. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

(A) $5/12$ **(B)** $1/11$ **(C)** $1/24$ **(D)** $1/22$ **(E)** $1/48$

Spring 2012 McNabb GDCTM Contest
Calculus

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. The value of $-3-3^2-3^3$ is equal to

   (A) -41 **(B)** -40 **(C)** -39 **(D)** -37 **(E)** -36

2. What is the largest possible product of two positive odd integers whose sum is 40?

   (A) 39 **(B)** 279 **(C)** 300 **(D)** 399 **(E)** 400

3. If $a \diamond b$ equals the lesser of $1/a$ and $1/b$, find the value of $-3 \diamond (-2 \diamond (-1/2))$.

   (A) -3 **(B)** -1/3 **(C)** -1/2 **(D)** -2 **(E)** -1

4. What is the remainder when the sum

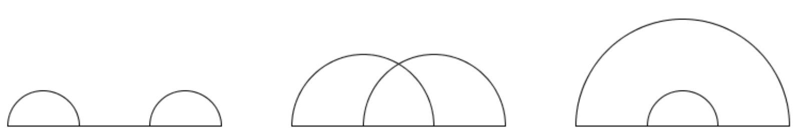
$$1^{111}+2^{111}+3^{111}+4^{111}+5^{111}+6^{111}+7^{111}+8^{111}+9^{111}+10^{111}$$

   is divided by 11?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

5. Distribute 14 points along a line segment. How many distinct ways exist for pairing the points via semicircles? The case of four points is pictured below.

   (A) 10395 **(B)** 40320 **(C)** 60125 **(D)** 101245 **(E)** 135135



6. If $2 + \ln x = \ln(x+2)$ then x must equal

(A) $\dfrac{2}{e^2-1}$ **(B)** $\dfrac{2}{e-1}$ **(C)** $\dfrac{1}{e^2-1}$ **(D)** $\dfrac{2}{e^2+1}$ **(E)** $\dfrac{1}{e^2+1}$

7. When expanded and simplified $(1 - x + x^2 - x^3)^{10}$ has the form

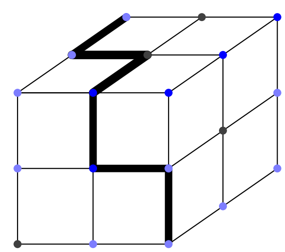
$$c_0 + c_1 x + c_2 x^2 + \cdots + c_{30} x^{30}$$

What is the value of $c_1 + c_3 + c_5 + \cdots + c_{29}$, the sum of the coefficients of all the odd powers of x?

(A) -4^{19} **(B)** -2^{20} **(C)** -2^{19} **(D)** 0 **(E)** 2^{19}

8. A small bug crawls on the surface of a $2 \times 2 \times 2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1 \times 1 \times 1$ cubes. How many shortest paths of this type are possible? One example is shown.

(A) 54 **(B)** 64 **(C)** 90 **(D)** 96 **(E)** 120



9. Find the value of $r^2s^2 + s^2t^2 + t^2r^2$ if r, s, and t are the three possibly complex roots of the cubic polynomial $x^3 + 5x^2 - 3x + 1$.

(A) -1 **(B)** 0 **(C)** 3 **(D)** 5 **(E)** 8

10. What is the sum

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{8} + \frac{1}{10} + \frac{1}{16} + \frac{1}{20} + \frac{1}{25} + \cdots$$

which consists of the sum of the reciprocals of 1 and all the natural numbers whose prime factorizations contain no primes other than 2 or 5?

(A) $5/2$ **(B)** $7/2$ **(C)** $5/3$ **(D)** 2 **(E)** 3

11. In $\triangle ABC$, the medians AD and BE are perpendicular. If $AC = 8$ and $BC = 12$, what is the length of AB?

(A) 6 **(B)** 9 **(C)** $4\sqrt{3}$ **(D)** 7 **(E)** $4\sqrt{13/5}$

12. When the polynomial x^{2012} is divided by the polynomial $x^2 + x + 1$ what is the remainder $R(x)$?

(A) 1 **(B)** $x + 1$ **(C)** $2x - 1$ **(D)** 0 **(E)** $-x - 1$

13. Which of the following are equal to $\int_a^b f(x)\,dx$ for all continuous functions f and all values of the constants a, b, and k, with $k \neq 0$?

I. $-\int_b^a f(x)\,dx$

II. $\int_a^b f(a+b-x)\,dx$

III. $\int_{ka}^{kb} f(kx)\,dx$.

(A) none of them **(B)** I only **(C)** I and II only
(D) I and III only **(E)** I, II and III

14. When the integrals listed below are arranged in order from least to greatest, which integral will be in the center?

$$\int_0^1 1 + x\,dx \qquad \int_0^1 \frac{1}{1+x}\,dx \qquad \int_0^1 \frac{1}{1+x+\frac{x^2}{2}}\,dx \qquad \int_0^1 e^x\,dx \qquad \int_0^1 e^{-x}\,dx$$

(A) $\int_0^1 1 + x\,dx$ **(B)** $\int_0^1 \frac{1}{1+x}\,dx$ **(C)** $\int_0^1 \frac{1}{1+x+\frac{x^2}{2}}\,dx$
(D) $\int_0^1 e^x\,dx$ **(E)** $\int_0^1 e^{-x}\,dx$

15. If $f(x)$ and $g(x)$ are differentiable functions and $F(x) = \int_0^{g(x)} f(t)\,dt$, then $F'(1)$ equals

(A) $f(g(1))$ **(B)** $f(1)g'(1)$ **(C)** $f'(g(1))g'(1)$ **(D)** $f(g(1))g'(1)$ **(E)** $f'(1)g'(1)$

16. Evaluate $\int_0^{49} \frac{1}{\sqrt{16-\sqrt{x}}}\,dx$.

(A) 28/3 **(B)** 32/3 **(C)** 12 **(D)** 40/3 **(E)** 44/3

17. Given that the area of an ellipse with semimajor axis a and semiminor axis b is πab, find the volume of the set of points $\{(x,y,z) : \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1\}$.

(A) 10π **(B)** 20π **(C)** 30π **(D)** 40π **(E)** 50π

18. John has just learned the arclength formula for functions of the form $y = f(x)$ and wishes to test this formula by measuring with a string the actual graph of $y = x^2$ from $x = 0$ to $x = 1$, with one unit in both the x and y direction measuring one inch. However, when he prints the graph of $y = x^2$ using his Computer Algebra System, he notes that while the unit in the x direction does measure exactly one inch, the unit in the y direction measures only three-quarters of an inch. Which integral below will give the length in inches of the actual printed curve $y = x^2$ from $x = 0$ to $x = 1$?

(A) $\int_0^1 \sqrt{1 + \frac{9}{4}x^2}\,dx$ **(B)** $\int_0^1 \sqrt{1 + \frac{4}{9}x^2}\,dx$ **(C)** $\int_0^1 \sqrt{1 + 4x^2}\,dx$
(D) $\int_0^1 \sqrt{1 + 9x^2}\,dx$ **(E)** $\int_0^1 \sqrt{1 + 6x^2}\,dx$

19. Given that $\ln 2 = 1 - 1/2 + 1/3 - 1/4 + 1/5 - 1/6 + \cdots$ and $\ln k = 1 + 1/2 + 1/3 - 3/4 + 1/5 + 1/6 + 1/7 - 3/8 + \cdots + 1/(4n+1) + 1/(4n+2) + 1/(4n+3) - 3/(4n+4) + \cdots$, then $k =$

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

20. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

(A) 5/12 **(B)** 1/11 **(C)** 1/24 **(D)** 1/22 **(E)** 1/48