Fall 2010 McNabb GDCTM Contest
Pre-Algebra

NO Calculators Allowed

1. Zeke's piggy bank has 111 coins. If it contains an equal number of quarters, dimes and nickels, and no other type of coin, the total value of the coins in his piggy bank is

   (A) \$14.80 **(B)** \$20.40 **(C)** \$24.00 **(D)** \$27.00 **(E)** \$44.40

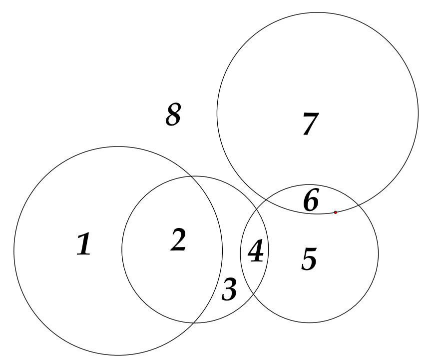
2. Keith's grandfather paid 70% of the cost of a jacket. Keith paid the rest. If Keith paid \$39 how much did his grandfather pay?

   (A) \$70 **(B)** \$91 **(C)** \$113 **(D)** \$130 **(E)** \$140

3. The remainder when 5^{2010} is divided by 7 equals

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 6

4. Four circles are drawn, all in the same plane. Find the maximum number of regions they can form. The diagram shows how four circles may form 8 regions.



   (A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

5. If the square root of a positive number falls between seven and eight, then the cube root of this number must fall between

   (A) 7 and 8 **(B)** 6 and 7 **(C)** 5 and 6
   (D) 4 and 5 **(E)** 3 and 4

6. What is the largest prime number which is a factor of every six digit number of the form $\underline{ABABAB}$?

   (A) 3 **(B)** 7 **(C)** 37 **(D)** 41 **(E)** 101

7. John has 54 coins totaling one dollar in value. Some are pennies; some are nickels; some are dimes. He has no other kind of coin. How many nickels does John have?

   (A) 7 **(B)** 9 **(C)** 11 **(D)** 13 **(E)** 15

8. How many lines of symmetry does a regular octagon have?

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

9. Seven consecutive integers are written on a whiteboard. When one of them is erased, the sum of the remaining six integers is 4208. What is the sum of the seven integers?

   (A) 4893 **(B)** 4900 **(C)** 4907 **(D)** 4914 **(E)** 4921

10. The next three Ranger batters get a hit with probabilities equal to 0.250, 0.300, and 0.500 respectively. What is the probability that all three get hits?

    (A) 1.000 **(B)** .300 **(C)** .125 **(D)** .05 **(E)** .0375

11. A positive integer has the interesting property that when expressed as a three digit base-7 number, those digits are the reverse of its digits when expressed as a base-9 number. What is this number expressed in normal form as a base-10 number?

    (A) 124 **(B)** 241 **(C)** 248 **(D)** 428 **(E)** 503

12. On a certain island, there are currently 1000 inhabitants, and 91% of these inhabitants were born there. Then some of these native inhabitants leave, so that now only 90% of the inhabitants of the island were born there. Assuming no other kind of change (births, deaths, immigration, etc...) in the population took place, how many of the native inhabitants left?

    (A) 9 **(B)** 10 **(C)** 40 **(D)** 90 **(E)** 100

13. How many positive integers less than 200 have an odd number of factors?

    (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 14

14. In how many ways can the the letters in *syzygy* be arranged so that the three y's do not all occur together?

(A) 96 **(B)** 112 **(C)** 113 **(D)** 114 **(E)** 120

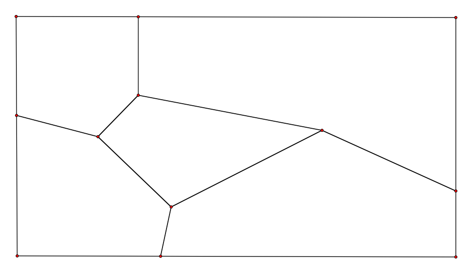
15. What is the greatest possible area of a triangle if two of its sides measure 8 and 13?

(A) 41 **(B)** 47 **(C)** 48 **(D)** 51 **(E)** 52

16. For her Math Club's fundraiser, Carla biked 8 miles south, then 8 miles west, and finally 7 miles south. How many miles, as the crow flies, was she from her start point?

(A) 14 **(B)** 15 **(C)** 17 **(D)** 20 **(E)** 23

17. A map maker has four colors available to color this map consisting of 5 counties. Each county is colored with a single color. No two counties that share a common boundary may be colored the same. In how many ways can our map maker color this map?



(A) 48 **(B)** 60 **(C)** 72 **(D)** 96 **(E)** 120

18. Points A, B, C, and D are collinear and occur in the same order as given. If the ratio $AB : BC$ equals 3 and the ratio $BD : AB$ equals $8/3$, then determine the ratio $CD : AC$.

(A) $\frac{5}{3}$ **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{7}{4}$ **(E)** $\frac{8}{3}$

19. What is the smallest possible product of three distinct numbers chosen from the set $\{-3, -2, -1, 0, 1, 3\}$

(A) -18 **(B)** -9 **(C)** -6 **(D)** 0 **(E)** 2

20. The height of a square pyramid is increased by 60% and the sides of its base are decreased by 20%. By what percent is the volume of the pyramid increased?

(A) 40% **(B)** 4.8% **(C)** 4% **(D)** 2.4% **(E)** 0%

21. A baseball team has won 50 games out of 75 so far played. If there are 45 games yet to be played, how many of these must be won in order for the team to finish its season having won exactly 60% of its games?

(A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

22. How many factors of 720 are also factors of 630?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

23. The value of the fraction

$$\frac{3+6+9+\cdots+99}{4+8+12+\cdots+132}$$

is

(A) 2/3 **(B)** 3/4 **(C)** 4/5 **(D)** 5/6 **(E)** 7

24. To specify the order of operations in multiplying 5 numbers together, three sets of parentheses are needed. Two ways, for example, are $((ab)(cd))e$ and $(((ab)c)d)e$. In how many ways can these three sets of parentheses be arranged? Assume the order of the numbers a through e is never changed.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

25. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

(A) 32 **(B)** 28 **(C)** 24 **(D)** 20 **(E)** 16

Fall 2010 McNabb GDCTM Contest
Algebra I

NO Calculators Allowed

1. The algebraic expression

$$(a-b-c)-(a+b-c)-(a-b+c)-(-a+b-c)$$

is equivalent to

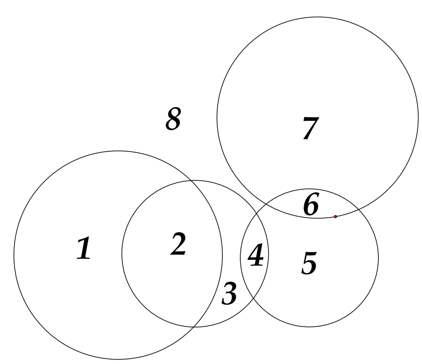
(A) $a+b+c$ **(B)** $-a$ **(C)** $-2a$ **(D)** $-2b$ **(E)** $b-c$

2. An automobile goes $y/9$ yards in d seconds. How many feet does it travel in two minutes time?

(A) $\frac{40y}{d}$ **(B)** $\frac{40d}{3y}$ **(C)** $\frac{3y}{40d}$ **(D)** $120yd$ **(E)** $\frac{120y}{d}$

3. The well-known formula $f = (9/5)c + 32$ relates the temperature f in Fahrenheit to the temperature c in Celsius. For how many values of f satisfying $32 \leq f \leq 212$, will the temperature be an integer in both of these scales?

(A) 9 **(B)** 10 **(C)** 19 **(D)** 20 **(E)** 21



4. Four circles are drawn, all in the same plane. Find the maximum number of regions they can form. The diagram shows how four circles may form 8 regions.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

5. If the square root of a positive number falls between seven and eight, then the cube root of this number must fall between

   (A) 7 and 8 **(B)** 6 and 7 **(C)** 5 and 6
   (D) 4 and 5 **(E)** 3 and 4

6. John has 54 coins totaling one dollar in value. Some are pennies; some are nickels; some are dimes. He has no other kind of coin. How many nickels does John have?

   (A) 7 **(B)** 9 **(C)** 11 **(D)** 13 **(E)** 15

7. A positive integer has the interesting property that when expressed as a three digit base-7 number, those digits are the reverse of its digits when expressed as a base-9 number. What is this number expressed in normal form as a base-10 number?

   (A) 124 **(B)** 241 **(C)** 248 **(D)** 428 **(E)** 503

8. On a certain island, there are currently 1000 inhabitants, and 91% of these inhabitants were born there. Then some of these native inhabitants leave, so that now only 90% of the inhabitants of the island were born there. Assuming no other kind of change (births, deaths, immigration, etc...) in the population took place, how many of the native inhabitants left?

   (A) 9 **(B)** 10 **(C)** 40 **(D)** 90 **(E)** 100

9. How many positive integers less than 200 have an odd number of factors?

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 14

10. At Zeke's Zucchini Stand, 3 zucchini's and 2 squash cost \$4.75, while 2 zucchini's and 3 squash cost \$5.25. How much would 3 zucchini's and 3 squash cost?

    (A) \$5.50 **(B)** \$5.75 **(C)** \$6 **(D)** \$6.25 **(E)** \$6.50

11. Two sides of a right triangle have lengths 6 and 8 respectively. The product of all the possible lengths of the third side can be written in the form $\sqrt{N}$, for some integer N. What is N?

(A) 100 **(B)** 2400 **(C)** 2800 **(D)** 3200 **(E)** 3600

12. In how many ways can the the letters in *syzygy* be arranged so that the three *y*'s do not all occur together?

(A) 96 **(B)** 112 **(C)** 113 **(D)** 114 **(E)** 120

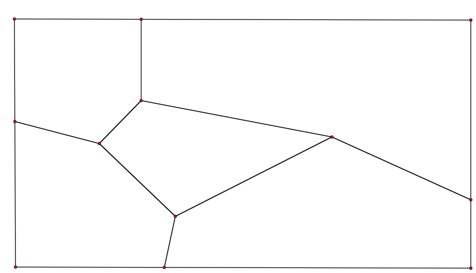
13. Points A, B, C, and D are collinear and occur in the same order as given. If the ratio $AB : BC$ equals 3 and the ratio $BD : AB$ equals $8/3$, then determine the ratio $CD : AC$.

(A) $\frac{5}{3}$ **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{7}{4}$ **(E)** $\frac{8}{3}$

14. A baseball team has won 50 games out of 75 so far played. If there are 45 games yet to be played, how many of these must be won in order for the team to finish its season having won exactly 60% of its games?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 72

15. A map maker has four colors available to color this map consisting of 5 counties. Each county is colored with a single color. No two counties that share a common boundary may be colored the same. In how many ways can our map maker color this map?



(A) 36 **(B)** 48 **(C)** 72 **(D)** 96 **(E)** 120

16. The value of the fraction

$$\frac{3+6+9+\cdots+99}{4+8+12+\cdots+132}$$

is

(A) 2/3 **(B)** 3/4 **(C)** 4/5 **(D)** 5/6 **(E)** 7

17. A bag contains 4 quarters and 2 dimes. If 3 coins are randomly removed from the bag, what is the expected total value in cents of these three coins?

(A) 50 **(B)** 55 **(C)** 60 **(D)** 65 **(E)** 75

18. The midpoints of the sides of a triangle are $(7,4)$, $(1,2)$, and $(1,6)$. What is the area of this triangle?

(A) 12 **(B)** 24 **(C)** 30 **(D)** 36 **(E)** 48

19. The set S contains seven numbers whose mean is 202. The mean of the four smallest numbers in S equals 100, while the mean of the four largest numbers in S equals 300. What is the median of all the numbers in S?

(A) 184 **(B)** 186 **(C)** 192 **(D)** 196 **(E)** 200

20. To specify the order of operations in multiplying 5 numbers together, three sets of parentheses are needed. Two ways, for example, are $((ab)(cd))e$ and $(((ab)c)d)e$. In how many ways can these three sets of parentheses be arranged? Assume the order of the numbers a through e is never changed.

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

21. Let $f(x) = |x+1| - |x| + |x-1|$. What is the minimum value of $f(x)$?

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

22. Let a, b, and c be positive integers with $LCM(a, b) = 48$, $LCM(b, c) = 42$, and $LCM(c, a) = 112$ then the value of $LCM(a, b, c)$ is

(A) 224 **(B)** 336 **(C)** 448 **(D)** 672 **(E)** cannot be determined

23. Find the area of quadrilateral $ABCD$ given that $AB = 13$, $BC = 10$, $CD = 10$, $DA = 13$, and $AC = 13$.

(A) 80 **(B)** 90 **(C)** 100 **(D)** 110 **(E)** 120

24. While Xerxes marched on Greece his army streched out for 50 miles. A dispatch rider had to ride from the rear to the head of the army, then instantly turn about and return to the rear. While he did this, the army advanced 50 miles. How many miles did the rider ride?

(A) 100 **(B)** 150 **(C)** $50 + 100\sqrt{2}$ **(D)** $100\sqrt{2}$ **(E)** $50 + 50\sqrt{2}$

25. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

(A) 32 **(B)** 28 **(C)** 24 **(D)** 20 **(E)** 16

Fall 2010 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. An automobile goes $y/9$ yards in d seconds. How many feet does it travel in two minutes time?

   (A) $\frac{40y}{d}$ **(B)** $\frac{40d}{3y}$ **(C)** $\frac{3y}{40d}$ **(D)** $120yd$ **(E)** $\frac{120y}{d}$

2. The well-known formula $f = (9/5)c + 32$ relates the temperature f in Fahrenheit to the temperature c in Celcius. For how many values of f satisfying $32 \leq f \leq 212$, will the temperature be an integer in both of these scales?

   (A) 9 **(B)** 10 **(C)** 19 **(D)** 20 **(E)** 21

3. Suppose the *converse* of the following statement is true:

   If Zerb is from Xanlor, then Zerb is blue.

   Which of the following statements **must** be true?

   I. If Zerb is not from Xanlor, then Zerb is not blue.
   II. If Zerb is from Xanlor, then Zerb is blue.
   III. If Zerb is blue, then Zerb is not from Xanlor.

   (A) I only **(B)** II only **(C)** III only
   (D) I and II only **(E)** I and III only

4. At Zeke's Zucchini Stand, 3 zucchini's and 2 squash cost \$4.75, while 2 zucchini's and 3 squash cost \$5.25. How much would 3 zucchini's and 3 squash cost?

   (A) \$5.50 **(B)** \$5.75 **(C)** \$6 **(D)** \$6.25 **(E)** \$6.50

5. A square is inscribed in a right triangle with sides of length 3, 4, and 5, so that one of the sides of the square is contained in the hypotenuse of the right triangle. What is the side length of the square?

(A) $\frac{60}{37}$ **(B)** 2 **(C)** $\frac{12}{5}$ **(D)** 3 **(E)** cannot be determined

6. In how many ways can the the letters in the string $ABECEDA$ be arranged so that the consonants are in alphabetical order?

(A) 90 **(B)** 105 **(C)** 120 **(D)** 180 **(E)** 210

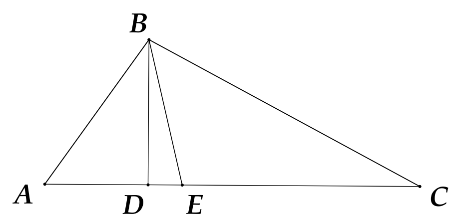
7. Points A, B, C, and D are collinear and occur in the same order as given. If the ratio $AB : BC$ equals 3 and the ratio $BD : AB$ equals $8/3$, then determine the ratio $CD : AC$.

(A) $\frac{5}{3}$ **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{7}{4}$ **(E)** $\frac{8}{3}$

8. A baseball team has won 50 games out of 75 so far played. If there are 45 games yet to be played, how many of these must be won in order for the team to finish its season having won exactly 60% of its games?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 72

9. In $\triangle ABC$, $\angle A = 60^\circ$, $\angle C = 40^\circ$, $BD \perp AC$ and $\overrightarrow{BE}$ bisects $\angle ABC$. Find the measure of $\angle DBE$ in degrees.



(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 20

10. A bag contains 4 quarters and 2 dimes. If 3 coins are randomly removed from the bag, what is the expected total value in cents of these three coins?

(A) 50 **(B)** 55 **(C)** 60 **(D)** 65 **(E)** 75

11. The midpoints of the sides of a triangle are $(7,4)$, $(1,2)$, and $(1,6)$. What is the area of this triangle?

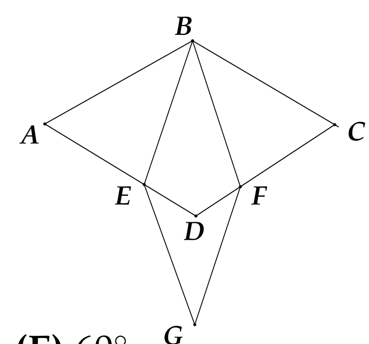
(A) 12 **(B)** 24 **(C)** 30 **(D)** 36 **(E)** 48

12. On co-planar lines l and m we choose points P_1, P_2, P_3, P_4, and P_5 on the former; and points Q_1, Q_2, Q_3, Q_4 on the latter. Draw all possible segments with one endpoint one of the P's and the other one of the Q's. What is the maximum total number of points that can be formed by intersection of pairs of these segments?

(A) 75 **(B)** 60 **(C)** 45 **(D)** 30 **(E)** 20

13. The set S contains seven numbers whose mean is 202. The mean of the four smallest numbers in S equals 100, while the mean of the four largest numbers in S equals 300. What is the median of all the numbers in S?

(A) 184 **(B)** 186 **(C)** 192 **(D)** 196 **(E)** 200



14. Quadrilaterals $ABCD$ and $BEGF$ are rhombi and are situated as in the diagram. If $\angle EBF = 20°$ and $\angle A = 50°$, what is $\angle DEG$?

(A) 40° **(B)** 45° **(C)** 50° **(D)** 55° **(E)** 60°

15. While Xerxes marched on Greece his army streched out for 50 miles. A dispatch rider had to ride from the rear to the head of the army, then instantly turn about and return to the rear. While he did this, the army advanced 50 miles. How many miles did the rider ride?

(A) 100 **(B)** $50+50\sqrt{2}$ **(C)** $100\sqrt{2}$ **(D)** 150 **(E)** $50+100\sqrt{2}$

16. How many non-congruent scalene triangles with integer side lengths exist with two sides of lengths 13 and 7 respectively?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

17. An isosceles trapezoid has bases of 11 and 21 units and legs of 13 units. What is the area of the trapezoid?

(A) 144 **(B)** 160 **(C)** 176 **(D)** 192 **(E)** 208

18. Hezy and Zeke were employed at different daily wages. At the end of a certain number of days Hezy received $300, while Zeke, who had been absent from work two of those days, received only $192. However, had it been the other way around, had Zeke worked all those days and Hezy been absent twice, then both would have received the same amount. What was Hezy's daily wage?

(A) 30 **(B)** 40 **(C)** 50 **(D)** 60 **(E)** 70

19. Suppose that the two legs of a certain right triangle are in the ratio 3 : 4. What is the greatest possible area of such a right triangle, if one of its altitudes measures 24?

(A) 216 **(B)** 384 **(C)** 486 **(D)** 600 **(E)** 726

20. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

(A) 32 **(B)** 28 **(C)** 24 **(D)** 20 **(E)** 16

Fall 2010 McNabb GDCTM Contest
Algebra II

NO Calculators Allowed

1. An automobile goes $y/9$ yards in d seconds. How many feet does it travel in two minutes time?

   (A) $\dfrac{40y}{d}$ **(B)** $\dfrac{40d}{3y}$ **(C)** $\dfrac{3y}{40d}$ **(D)** $120yd$ **(E)** $\dfrac{120y}{d}$

2. The well-known formula $f = (9/5)c + 32$ relates the temperature f in Fahrenheit to the temperature c in Celcius. For how many values of f satisfying $32 \leq f \leq 212$, will the temperature be an integer in both of these scales?

   (A) 9 **(B)** 10 **(C)** 19 **(D)** 20 **(E)** 21

3. If $f(\sqrt{x}) = \dfrac{x+3}{13}$ and $g(x^2) = x^4 + 3x^2 - 22$, then find $f(g(4))$.

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

4. A square is inscribed in a right triangle with sides of length 3, 4, and 5, so that one of the sides of the square is contained in the hypotenuse of the right triangle. What is the side length of the square?

   (A) $\dfrac{60}{37}$ **(B)** 2 **(C)** $\dfrac{12}{5}$ **(D)** 3 **(E)** cannot be determined

5. The arithmetic mean of a, b, and c is 7 and the arithmetic mean of a^2, b^2, and c^2 is 55. What is the arithmetic mean of ab, bc, and ac?

   (A) 24 **(B)** 31 **(C)** 46 **(D)** 48 **(E)** 92

6. In how many ways can the the letters in the string $ABECEDA$ be arranged so that the consonants are in alphabetical order?

   (A) 90 **(B)** 105 **(C)** 120 **(D)** 180 **(E)** 210

7. The graph of the quadratic function $f(x) = ax^2 + bx + c$ contains the points $(-1, 6)$, $(7, 6)$, and $(1, -6)$. What is the minimum value of $f(x)$?

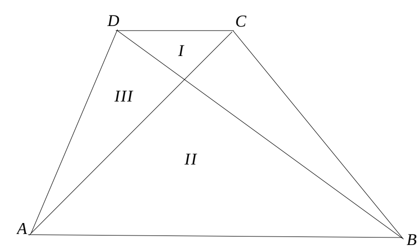
(A) -36 **(B)** -26 **(C)** -20 **(D)** -10 **(E)** -6

8. Let a, b, and c be positive real numbers. Supposing that $ab = kc$, $ac = lb$, and $bc = ma$, then c must equal

(A) lm **(B)** $\sqrt{klm}$ **(C)** $\sqrt{\frac{l}{m}}$ **(D)** $k\sqrt{lm}$ **(E)** $\sqrt{lm}$

9. The real number $\sqrt{16 + \sqrt{220}}$ can be expressed in the form $\sqrt{A} + \sqrt{B}$, where A and B are integers and $A > B$. What is the value of $A - B$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10



10. In trapezoid $ABCD$, the area of region I is 9 and the area of region II is 16. What is the area of region III?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 12.5 **(E)** cannot be determined

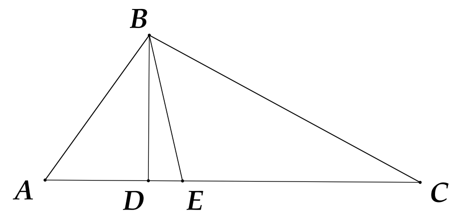
11. The polynomial

$$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

can be factored as $P_2(x) \cdot P_6(x)$ where P_2 and P_6 are polynomials with integer coefficients and have degrees 2 and 6 respectively. Find the sum of the coefficients of P_2.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

12. In $\triangle ABC$, $\angle A = 60^\circ$, $C = 40^\circ$, $BD \perp AC$, and $\overrightarrow{BE}$ bisects $\angle ABC$. Find the measure of $\angle DBE$ in degrees.



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(A) 184 **(B)** 186 **(C)** 192 **(D)** 196 **(E)** 200

15. Let a and b be positive constants. If x is a solution of

$$\sqrt{x+a} + \sqrt{x+b} = \sqrt{a+b}$$

then x must equal

(A) 0 **(B)** $\frac{a+b}{2}$ **(C)** $-\frac{ab}{a+b}$
(D) $-\left(\frac{1}{a}+\frac{1}{b}\right)$ **(E)** $-\frac{2}{a+b}$

16. While Xerxes marched on Greece his army streched out for 50 miles. A dispatch rider had to ride from the rear to the head of the army, then instantly turn about and return to the rear. While he did this, the army advanced 50 miles. How many miles did the rider ride?

(A) 100 **(B)** $50+50\sqrt{2}$ **(C)** $100\sqrt{2}$
(D) 150 **(E)** $50+100\sqrt{2}$

17. The sum of two of the roots of $p(x) = 4x^3 + 8x^2 - 9x - k$, where k is a constant, is zero. Find the value of k.

(A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 200

18. Let f be a function such that $f(x+y) = f(xy)$ for all real numbers x and y. If it is also known that $f(5) = 5$, determine the value of $f(25)$.

(A) 1 **(B)** 5 **(C)** 10 **(D)** 20 **(E)** 25

19. Hezy and Zeke were employed at different daily wages. At the end of a certain number of days Hezy received \$300, while Zeke, who had been absent from work two of those days, received only \$192. However, had it been the other way around, had Zeke worked all those days and Hezy been absent twice, then both would have received the same amount. What was Hezy's daily wage?

(A) 30 **(B)** 40 **(C)** 50 **(D)** 60 **(E)** 70

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Pre-Calculus

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   (A) 9 **(B)** 10 **(C)** 19 **(D)** 20 **(E)** 21

2. A positive integer has the interesting property that when expressed as a three digit base-7 number, those digits are the reverse of its digits when expressed as a base-9 number. What is this number expressed in normal form as a base-10 number?

   (A) 124 **(B)** 241 **(C)** 248 **(D)** 428 **(E)** 503

3. What is the maximum number of times that the graph of a polynomial of degree six can intersect the graph of a polynomial of degree five?

   (A) 1 **(B)** 5 **(C)** 6 **(D)** 11 **(E)** 30

4. The arithmetic mean of a, b, and c is 7 and the arithmetic mean of a^2, b^2, and c^2 is 55. What is the arithmetic mean of ab, bc, and ac?

   (A) 24 **(B)** 31 **(C)** 46 **(D)** 48 **(E)** 92

5. Given the arithmetic series

$$S = 3 + 6 + 9 + \cdots + 99$$
$$T = 4 + 8 + 12 + \cdots + 132$$

   what is the value of the ratio $S : T$?

   (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{5}$ **(E)** $\frac{5}{6}$

6. A square is inscribed in a right triangle with sides of length 3, 4, and 5, so that one of the sides of the square is contained in the hypotenuse of the right triangle. What is the side length of the square?

(A) $\frac{60}{37}$ **(B)** 2 **(C)** $\frac{12}{5}$ **(D)** 3 **(E)** cannot be determined

7. In how many ways can the the letters in the string $ABECEDA$ be arranged so that the consonants are in alphabetical order?

(A) 90 **(B)** 105 **(C)** 120 **(D)** 180 **(E)** 210

8. Hezy and Zeke were employed at different daily wages. At the end of a certain number of days Hezy received $300, while Zeke, who had been absent from work two of those days, received only $192. However, had it been the other way around, had Zeke worked all those days and Hezy been absent twice, then both would have received the same amount. What was Hezy's daily wage?

(A) 30 **(B)** 40 **(C)** 50 **(D)** 60 **(E)** 70

9. The real number $\sqrt{16+\sqrt{220}}$ can be expressed in the form $\sqrt{A}+\sqrt{B}$, where A and B are integers and $A > B$. What is the value of $A - B$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

10. What is the slope of the line that bisects the acute angle formed by the lines $y = (5/12)x$ and $y = (3/4)x$?

(A) $\frac{1}{2}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{2}{3}$

11. Let A and B satisfy $\log_2(\log_4 A) = 1$ and $\log_4(\log_2 B) = 1$. Find the value of $\frac{1}{A} + \frac{1}{B}$.

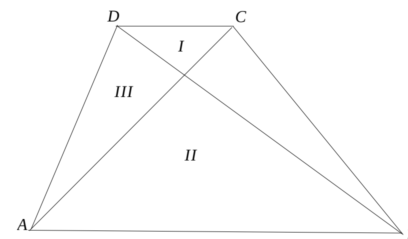
(A) 2 **(B)** 1 **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{16}$

12. The value of $\sin 195^\circ + \sin 105^\circ$ is equal to

(A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{2}}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** 1 **(E)** $\sqrt{2}$

13. The smallest positive integer which can be expressed both as a sum of ten consecutive positive integers and eleven consecutive positive integers is

(A) 55 **(B)** 110 **(C)** 130 **(D)** 165 **(E)** 275



14. In trapezoid $ABCD$, the area of region I is 9 and the area of region II is 16. What is the area of region III?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 12.5 **(E)** cannot be determined

15. The polynomial

$$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

can be factored as $P_2(x) \cdot P_6(x)$ where P_2 and P_6 are polynomials with integer coefficients and have degrees 2 and 6 respectively. Find the sum of the coefficients of P_2.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

16. Given $\triangle ABC$ with $AB = AC = 10$, points D, E, and F lie on sides BC, AC, and AB respectively so that $BF = 5$, $EC = 4$, and the three segments AD, BE, and CF are concurrent. If this point of concurrency is named O, what is the ratio $AO : OD$?

(A) $\frac{5}{2}$ **(B)** $\frac{11}{4}$ **(C)** $\frac{14}{5}$ **(D)** 3 **(E)** $\frac{7}{2}$

17. The sum of two of the roots of $p(x) = 4x^3 + 8x^2 - 9x - k$, where k is a constant, is zero. Find the value of k.

(A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 200

18. An urn contains two red, two blue, two white, and two yellow balls. Susan draws balls at random from the urn without replacing them. What is the expected number of draws Susan makes until drawing her first red ball?

(A) $\frac{42}{14}$ **(B)** $\frac{43}{14}$ **(C)** $\frac{42}{13}$ **(D)** $\frac{43}{13}$ **(E)** $\frac{44}{13}$

19. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

(A) 32 **(B)** 28 **(C)** 24 **(D)** 20 **(E)** 16

20. In $\triangle ABC$, $AB = 5$, $BC = 6$, and $CA = 4$. Side BC is trisected by points P and Q. Determine the value of $(AP)^2 + (AQ)^2$.

(A) $7\sqrt{7}$ **(B)** 25 **(C)** 29 **(D)** $11\sqrt{7}$ **(E)** 33

Fall 2010 McNabb GDCTM Contest
Calculus

NO Calculators Allowed

1. How many non-congruent scalene triangles with integer side lengths exist with two sides of lengths 13 and 7 respectively?

   (A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

2. Let A and B satisfy $\log_2(\log_4 A) = 1$ and $\log_4(\log_2 B) = 1$. Find the value of $\frac{1}{A} + \frac{1}{B}$.

   (A) 2 **(B)** 1 **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{16}$

3. The value of $\sin 195^\circ + \sin 105^\circ$ is equal to

   (A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{2}}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** 1 **(E)** $\sqrt{2}$

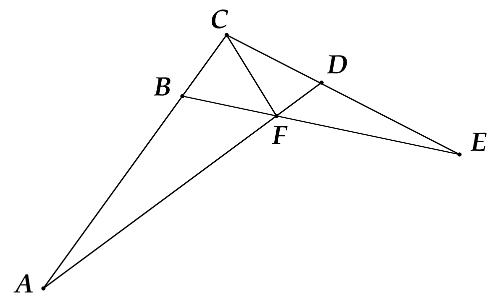
4. A positive integer has the interesting property that when expressed as a three digit base-7 number, those digits are the reverse of its digits when expressed as a base-9 number. What is this number expressed in normal form as a base-10 number?

   (A) 124 **(B)** 241 **(C)** 248 **(D)** 428 **(E)** 503

5. In how many ways can the the letters in the string $ABECEDA$ be arranged so that the consonants are in alphabetical order?

   (A) 90 **(B)** 105 **(C)** 120 **(D)** 180 **(E)** 210

6. In the configuration shown, the area of $\triangle ABF$ is 11, the area of $\triangle CFD$ is 3, and the area of $\triangle DEF$ is $11/3$. Find the area of $\triangle BCF$.



(A) $\dfrac{11}{4}$ **(B)** 3 **(C)** 4 **(D)** $\dfrac{14}{3}$ **(E)** 8

7. Hezy and Zeke were employed at different daily wages. At the end of a certain number of days Hezy received \$300, while Zeke, who had been absent from work two of those days, received only \$192. However, had it been the other way around, had Zeke worked all those days and Hezy been absent twice, then both would have received the same amount. What was Hezy's daily wage?

(A) 30 **(B)** 40 **(C)** 50 **(D)** 60 **(E)** 70

8. What is the slope of the line that bisects the acute angle formed by the lines $y = (5/12)x$ and $y = (3/4)x$?

(A) $\dfrac{1}{2}$ **(B)** $\dfrac{7}{12}$ **(C)** $\dfrac{5}{8}$ **(D)** $\dfrac{4}{7}$ **(E)** $\dfrac{2}{3}$

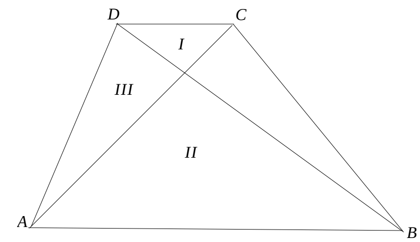
9. The sum of two of the roots of $p(x) = 4x^3 + 8x^2 - 9x - k$, where k is a constant, is zero. Find the value of k.

(A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 200

10. An urn contains two red, two blue, two white, and two yellow balls. Susan draws balls at random from the urn without replacing them. What is the expected number of draws Susan makes until drawing her first red ball?

(A) $\dfrac{42}{14}$ **(B)** $\dfrac{43}{14}$ **(C)** $\dfrac{42}{13}$ **(D)** $\dfrac{43}{13}$ **(E)** $\dfrac{44}{13}$

11. In trapezoid $ABCD$, the area of region I is 9 and the area of region II is 16. What is the area of region III?



(A) 10 **(B)** 11 **(C)** 12 **(D)** 12.5 **(E)** cannot be determined

12. The polynomial

$$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

can be factored as $P_2(x) \cdot P_6(x)$ where P_2 and P_6 are polynomials with integer coefficients and have degrees 2 and 6 respectively. Find the sum of the coefficients of P_2.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

13. The number of ordered pairs of integers that satisfy the equation $x^2 + 4x + y^2 = 9$ is

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

14. Let a and b be positive constants. If x is a solution of

$$\sqrt{x+a} + \sqrt{x+b} = \sqrt{a+b}$$

then x must equal

(A) 0 **(B)** $\dfrac{a+b}{2}$ **(C)** $-\dfrac{ab}{a+b}$ **(D)** $-\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$ **(E)** $-\dfrac{2}{a+b}$

15. Which of the following lines is an asymptote of the curve $x^2 - 4xy + 3y^2 = 7$?

(A) $x + 3y = 0$ **(B)** $x - 3y = 0$ **(C)** $x + y = 0$ **(D)** $x + y = -3$ **(E)** $x - y = 1$

16. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

(A) 32 **(B)** 28 **(C)** 24 **(D)** 20 **(E)** 16

17. Evaluate the following limit:

$$\lim_{x\to\infty} \left(x^3 + ax^2\right)^{1/3} - \left(x^3 - ax^2\right)^{1/3}$$

(A) 0 **(B)** 1 **(C)** $\frac{a}{3}$ **(D)** $\frac{2a}{3}$ **(E)** a

18. In the piecewise function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{1}{1+x^2} & \text{if } x \leq 1 \\ a(x-1)^2 + b(x-1) + c & \text{if } x > 1 \end{cases}$$

the constants a, b, and c are chosen to ensure that f is twice differentiable over the real numbers. What then must be the value of a?

(A) -2 **(B)** -1 **(C)** $-\frac{1}{2}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{2}$

19. Let $f(x)$ be differentiable with $f' > 0$. Let $g = f^{-1}$. Find the value of $(g \circ g)'(3)$ if $f(4) = 3$, $f'(4) = 2$, $f(5) = 4$, and $f'(5) = 5$.

(A) $\frac{1}{10}$ **(B)** $\frac{1}{5}$ **(C)** 2 **(D)** 5 **(E)** 20

20. Let g be 10 times differentiable with $g(0) = 1/8!$. Suppose $f(x) = x^{10}g(x)$. Find $f^{(10)}(0)$.

(A) $\frac{1}{8!}$ **(B)** 1 **(C)** 90 **(D)** 120 **(E)** 8!