Spring 2010 McNabb GDCTM Contest
Level I

1. Three pomegranates and one pineapple weigh as much as sixteen plums. Four plums and one pomegranate weight as much as one pineapple. How many pomegranates weigh as much as 3 pineapples?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

2. What is the area of the quadrilateral in the coordinate plane with vertices whose coordinates are (in order): $(0,0)$, $(7,1)$, $(4,4)$, and $(2,11)$?

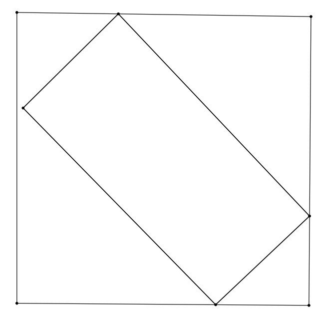
   (A) 30 **(B)** 31 **(C)** 31.5 **(D)** 32 **(E)** 33

3. Ronald has an unlimited number of 5 cent and 7 cent stamps. What is the largest amount of postage (in cents) that he cannot make with these stamps?

   (A) 16 **(B)** 22 **(C)** 23 **(D)** 79 **(E)** 99

4. A rectangle with unequal sides is placed in a square so that each vertex of the rectangle lies on a side of the square at a trisection point of that side as shown. What is the fraction of the area of the square that is covered by the rectangle?

   (A) 1/3 **(B)** 7/18 **(C)** 4/9 **(D)** 1/2 **(E)** 5/9



5. The area of a triangle with sides of length 13, 14, and 15 is closest to

   (A) 84 **(B)** 86 **(C)** 88 **(D)** 90 **(E)** 92

6. If $f(x)$ is a linear function for which $f(8) - f(1) = 11$, then $f(41) - f(6)$ is equal to

   (A) 61 **(B)** 55 **(C)** 49 **(D)** 43 **(E)** 37

7. The surface area of a large spherical balloon is doubled. By what factor is the volume of the balloon increased?

   (A) 8 **(B)** 4 **(C)** $2\sqrt{2}$ **(D)** $\sqrt[3]{4}$ **(E)** 2

8. Find the distance between the point with coordinates $(14, -2)$ and the line with equation $3x - 4y = 0$.

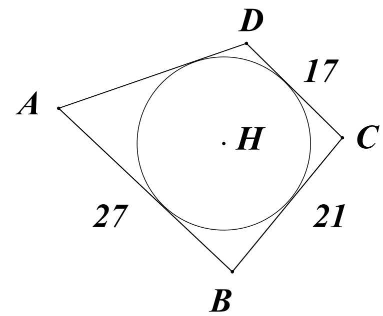
   (A) 4 **(B)** $4\sqrt{2}$ **(C)** $5\sqrt{2}$ **(D)** 8 **(E)** 10

9. Zeke cycles steadily for 36 miles. If he had managed to go 3 mph faster, he would have taken one hour less for the trip. What was Zeke's actual speed in mph during this trip?

(A) 9 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 21

10. A circle is inscribed in quadrilateral $ABCD$ as marked. Find the length of side $\overline{DA}$.

(A) 23 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27



11. How many lines of symmetry does a cube have?

(A) 4 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 13

12. In $\triangle ABC$, let D be the intersection point of the bisector of $\angle ABC$ and the bisector of $\angle BCA$. If $\angle CAB$ is 70 degrees, what is the measure of $\angle CDB$ in degrees?

(A) 35 **(B)** 55 **(C)** 105 **(D)** 125 **(E)** 140

13. A set of seven distinct positive integers has a mean of 13. Find the difference between the greatest possible median of these integers and the least possible median of these integers.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

14. A line L in the coordinate plane has slope -2. Suppose the triangle with vertices given by the origin, the x-intercept of L, and the y-intercept of L has area 9. Then an equation for L could be

(A) $2x + y = 0$ **(B)** $2x + y = 4$ **(C)** $-2x + y = 6$
(D) $2x + y = 3$ **(E)** $2x + y = -6$

15. Each vertex of a cube is randomly colored red or blue with each color being equally likely. What is the probability that every pair of adjacent vertices have different colors?

(A) 0 **(B)** 1/128 **(C)** 1/64 **(D)** 1/32 **(E)** 1/2

16. A semicircle lies in $\triangle EFG$ with diameter contained in $\overline{EG}$, and with $\overline{EF}$ and $\overline{GF}$ both tangent to it. If $EF = 12$, $FG = 15$, and $EG = 18$, what is the value of EC where C is the center of the semicircle?

(A) 6 **(B)** 6.5 **(C)** 7 **(D)** 7.5 **(E)** 8

17. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1 inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(A) 576 **(B)** 864 **(C)** 1152 **(D)** 1200 **(E)** 1600

18. In acute $\triangle ABC$, the altitude from A meets side $\overline{BC}$ at point D, the altitude from B meets side $\overline{AC}$ at point E, and the altitude from C meets side $\overline{AB}$ at point F. All three altitudes are concurrent at point H lying inside $\triangle ABC$. If $\angle BAC$ measures 58 degrees, then find the measure of $\angle BHC$ in degrees.

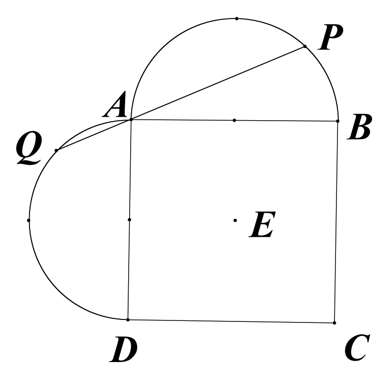
(A) 90 **(B)** 98 **(C)** 104 **(D)** 116 **(E)** 122

19. In how many ways can five distinct books be arranged in a bookcase with 3 shelves, each shelf capable of holding all five books?

(A) 19 **(B)** 120 **(C)** 360 **(D)** 840 **(E)** 2520

20. Semicircles are drawn on two sides of square $ABCD$ as shown. Point E is the center of the square, and points Q, A, and P are collinear with $QA = 4$ and $AP = 16$. Find QE.

(A) 12 **(B)** $10\sqrt{2}$ **(C)** $10\sqrt{3}$
(D) 15 **(E)** 20



Spring 2010 McNabb GDCTM Contest
Level II

1. What is the area of the quadrilateral in the coordinate plane with vertices whose coordinates are (in order): $(0, 0)$, $(7, 1)$, $(4, 4)$, and $(2, 11)$?

   (A) 30 **(B)** 31 **(C)** 31.5 **(D)** 32 **(E)** 33

2. Ronald has an unlimited number of 5 cent and 7 cent stamps. What is the largest amount of postage (in cents) that he cannot make with these stamps?

   (A) 16 **(B)** 22 **(C)** 23 **(D)** 79 **(E)** 99

3. Find the distance between the point with coordinates $(14, -2)$ and the line with equation $3x - 4y = 0$.

   (A) 4 **(B)** $4\sqrt{2}$ **(C)** $5\sqrt{2}$ **(D)** 8 **(E)** 10

4. Zeke cycles steadily for 36 miles. If he had managed to go 3 mph faster, he would have taken one hour less for the trip. What was Zeke's actual speed in mph during this trip?

   (A) 9 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 21

5. How many lines of symmetry does a cube have?

   (A) 4 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 13

6. In $\triangle ABC$, let D be the intersection point of the bisector of $\angle ABC$ and the bisector of $\angle BCA$. If $\angle CAB$ is 70 degrees, what is the measure of $\angle CDB$ in degrees?

   (A) 35 **(B)** 55 **(C)** 105 **(D)** 125 **(E)** 140

7. Hezy eats y yogurts every d days. How many yogurts does he eat in w weeks?

   (A) $\frac{7yw}{d}$ **(B)** $\frac{7w}{yd}$ **(C)** $\frac{yd}{7w}$ **(D)** $7dwy$ **(E)** $\frac{7yd}{w}$

8. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

   (A) $\frac{5}{72}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{16}$ **(E)** $\frac{3}{8}$

9. A set of seven distinct positive integers has a mean of 13. Find the difference between the greatest possible median of these integers and the least possible median of these integers.

   (A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

10. The parabola $y = ax^2 + bx + c$ passes through the points $(-2, 3)$, $(2, -1)$, and $(6, 12)$. The value of the coefficient a equals

(A) $1/4$ **(B)** $3/16$ **(C)** $5/16$ **(D)** $17/32$ **(E)** $1/2$

11. The *centroid* of a triangle is the point of concurrence of its medians. In the $x - y$ plane point A has coordinates $(0, 0)$, point B has coordinates $(5, 15)$, and point C has coordinates $(13, 9)$. The line p passes through the point B and the centroid of $\triangle ABC$. Another point on line p is

(A) $(6, 9)$ **(B)** $(12, -2)$ **(C)** $(7, 1)$ **(D)** $(0, 43)$ **(E)** $(-4, -4)$

12. If $\sin x + \cos x = a$, then $\sin 2x$ equals

(A) $2a$ **(B)** $a^2 - 1$ **(C)** $1 - a^2$ **(D)** $a^2 + 1$ **(E)** $(a - 1)^2$

13. A line L in the coordinate plane has slope -2. Suppose the triangle with vertices given by the origin, the x-intercept of L, and the y-intercept of L has area 9. Then an equation for L could be

(A) $2x + y = 0$ **(B)** $2x + y = 4$ **(C)** $-2x + y = 6$
(D) $2x + y = 3$ **(E)** $2x + y = -6$

14. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1 inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(A) 576 **(B)** 864 **(C)** 1152 **(D)** 1200 **(E)** 1600

15. In acute $\triangle ABC$, the altitude from A meets side $\overline{BC}$ at point D, the altitude from B meets side $\overline{AC}$ at point E, and the altitude from C meets side $\overline{AB}$ at point F. All three altitudes are concurrent at point H lying inside $\triangle ABC$. If $\angle BAC$ measures 58 degrees, then find the measure of $\angle BHC$ in degrees.

(A) 90 **(B)** 98 **(C)** 104 **(D)** 116 **(E)** 122

16. In how many ways can five distinct books be arranged in a bookcase with 3 shelves, each shelf capable of holding all five books?

(A) 19 **(B)** 120 **(C)** 360 **(D)** 840 **(E)** 2520

17. Two ferries start at the same instant from opposite banks of a river. They travel directly across the river. Each boat keeps its own constant speed, though one boat is faster than the other. In this first trip across they pass at a point 720 yards from the nearer bank. When reaching the opposite shore each boat remains exactly 10 minutes in its dock before heading back the other way. On this trip back the boats meet 400 yards from the other shore. How wide is the river (in yards)?

(A) 1040 **(B)** 1120 **(C)** 1520 **(D)** 1600 **(E)** 1760

18. If p and q are integers and

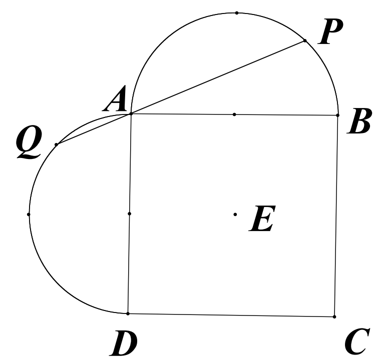
$$p\log_{200} 5 + q\log_{200} 2 = 3$$

then determine the value of $p+q$.

(A) 10 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 20

19. Semicircles are drawn on two sides of square $ABCD$ as shown. Point E is the center of the square, and points Q, A, and P are collinear with $QA = 4$ and $AP = 16$. Find QE.

(A) 12 **(B)** $10\sqrt{2}$ **(C)** $10\sqrt{3}$
(D) 15 **(E)** 20



20. In $\triangle ABC$, points D, E, and F are located on $\overline{BC}$, $\overline{AC}$, and $\overline{BA}$ respectively, so that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are concurrent at point P, the area of $\triangle BPD$ is 190, the area of $\triangle DPC$ is 380, and the area of $\triangle CPE$ is 418. Then the area of $\triangle APE$ is

(A) 121 **(B)** 143 **(C)** 242 **(D)** 319 **(E)** 330

Spring 2010 McNabb GDCTM Contest
Level III

1. Amy is in a line of 40 people at the movies. Ahead of her stand 12 people. How many stand behind her?

   (A) 13 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 39

2. Abigail, Brice, and Carl all start out with some one-dollar bills in their pockets. In particular, Carl starts out with 4 one-dollar bills. Brice then gives half of his dollar bills to Abigail and the other half to Carl. Then Abigail gives half of her bills to Brice and the other half to Carl. Finally, Carl gives half of his bills to Abigail and keeps the rest. If now, at the end of these exchanges, Abigail, Brice, and Carl all have 8 one-dollar bills, how many such bills did Abigail have to begin with?

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

3. Let a, b, c, and d be positive real numbers such that $abc = 12$, $bcd = 6$, $acd = 125$, and $abd = 3$. Then $abcd$ equals

   (A) 12 **(B)** 15 **(C)** 30 **(D)** 45 **(E)** 60

4. Let $f(x)$ be a linear function for which $f(8) - f(1) = 11$. Then $f(41) - f(6)$ equals

   (A) 37 **(B)** 43 **(C)** 49 **(D)** 55 **(E)** 61

5. Three pomegranates and one pineapple weigh as much as sixteen plums. Four plums and one pomegranate weight as much as one pineapple. How many pomegranates weigh as much as 3 pineapples?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

6. If p and q are integers and
$$p \log_{200} 5 + q \log_{200} 2 = 3$$
   then determine the value of $p + q$.

   (A) 10 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 20

7. For $z = a + bi$ a complex number, it's conjugate is $\overline{z} = a - bi$. Let S denote the set of all complex numbers z so that the real part of $1/\overline{z}$ equals one. Then set S is

   (A) a line **(B)** a circle **(C)** a parabola
   (D) the empty set **(E)** an hyperbola

8. If a is a multiple of 14 and b is a multiple of 21, then what is the largest integer that must be a factor of any integer of the form $9a + 8b$?

(A) 84 **(B)** 42 **(C)** 21 **(D)** 14 **(E)** 8

9. How many lines of symmetry does a cube have?

(A) 4 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 13

10. The parabola $y = ax^2 + bx + c$ passes through the points $(-2, 3)$, $(2, -1)$, and $(6, 12)$. The value of the coefficient a equals

(A) 1/4 **(B)** 3/16 **(C)** 5/16 **(D)** 17/32 **(E)** 1/2

11. The *centroid* of a triangle is the point of concurrence of its medians. In the $x - y$ plane point A has coordinates $(0, 0)$, point B has coordinates $(5, 15)$, and point C has coordinates $(13, 9)$. The line p passes through the point B and the centroid of $\triangle ABC$. Another point on line p is

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12. If $\sin x + \cos x = a$, then $\sin 2x$ equals

(A) $2a$ **(B)** $a^2 - 1$ **(C)** $1 - a^2$ **(D)** $a^2 + 1$ **(E)** $(a - 1)^2$

13. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

(A) $\frac{5}{72}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{16}$ **(E)** $\frac{3}{8}$

14. The function $f(x) = \dfrac{x + \sqrt{x^2 + 8}}{2}$ has domain $(-\infty, \infty)$ and on this domain is invertible. It's inverse function has the form $f^{-1}(x) = ax + \dfrac{b}{x}$ for some constants a and b. What is the value of $a + b$?

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

15. If $x + \dfrac{1}{x} = 3$ then what is the value of $\dfrac{x^4 + 1}{x^2}$?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

16. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(A) 576 **(B)** 864 **(C)** 1152 **(D)** 1200 **(E)** 1600

17. A semicircle lies in $\triangle EFG$ with diameter contained in $\overline{EG}$, and with $\overline{EF}$ and $\overline{GF}$ both tangent to it. If $EF = 12$, $FG = 15$, and $EG = 18$, what is the value of EC where C is the center of the semicircle?

(A) 6 **(B)** 6.5 **(C)** 7 **(D)** 7.5 **(E)** 8

18. A cubic polynomial $P(x)$ satisfies $P(1) = 1$, $P(2) = 3$, $P(3) = 5$, and $P(4) = 6$. Then the value $P(7)$ must equal

(A) 10 **(B)** 7 **(C)** 0 **(D)** -3 **(E)** -7

19. Hezy tosses a fair cubical die until each number 1 through 6 has come up at least once. On average, Hezy requires X tosses to do this. The real number X is closest to

(A) 12.8 **(B)** 14.7 **(C)** 16.3 **(D)** 17.2 **(E)** 19.5

20. In $\triangle ABC$, points D, E, and F are located on $\overline{BC}$, $\overline{AC}$, and $\overline{BA}$ respectively, so that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are concurrent at point P, the area of $\triangle BPD$ is 190, the area of $\triangle DPC$ is 380, and the area of $\triangle CPE$ is 418. Then the area of $\triangle APE$ is

(A) 121 **(B)** 143 **(C)** 242 **(D)** 319 **(E)** 330

Spring 2010 McNabb GDCTM Contest
Level IV

1. Amy is in a line of 40 people at the movies. Ahead of her stand 12 people. How many stand behind her?

   (A) 13 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 39

2. Let a, b, c, and d be positive real numbers such that $abc = 12$, $bcd = 6$, $acd = 125$, and $abd = 3$. Then $abcd$ equals

   (A) 12 **(B)** 15 **(C)** 30 **(D)** 45 **(E)** 60

3. Three pomegranates and one pineapple weigh as much as sixteen plums. Four plums and one pomegranate weight as much as one pineapple. How many pomegranates weigh as much as 3 pineapples?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

4. For $z = a + bi$ a complex number, it's conjugate is $\overline{z} = a - bi$. Let S denote the set of all complex numbers z so that the real part of $1/\overline{z}$ equals one. Then set S is

   (A) a line **(B)** a circle **(C)** a parabola
   (D) the empty set **(E)** an hyperbola

5. How many lines of symmetry does a cube have?

   (A) 4 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 13

6. If $\sin x + \cos x = a$, then $\sin 2x$ equals

   (A) $2a$ **(B)** $a^2 - 1$ **(C)** $1 - a^2$ **(D)** $a^2 + 1$ **(E)** $(a - 1)^2$

7. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

   (A) $\frac{5}{72}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{16}$ **(E)** $\frac{3}{8}$

8. The function $f(x) = \dfrac{x + \sqrt{x^2 + 8}}{2}$ has domain $(-\infty, \infty)$ and on this domain is invertible. It's inverse function has the form $f^{-1}(x) = ax + \dfrac{b}{x}$ for some constants a and b. What is the value of $a + b$?

   (A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

9. A regular pentagon has each edge of length 2. Its area is closest to

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

10. Let k be a positive constant and let f be a continuous function on the interval $[-k, k]$. If $\int_{-k}^{k} f(x)\,dx = a$ then $\int_{-1}^{1} f(kx)\,dx$ equals

(A) a **(B)** ak **(C)** 1 **(D)** $\dfrac{k}{a}$ **(E)** $\dfrac{a}{k}$

11. Let f be continuously differentiable on the interval $[0, \pi]$. If $f(0) = 0$ and $f(\pi) = 0$, then

$$\int_0^{\pi} f(x)f'(x)\,dx$$

equals

(A) $-\pi$ **(B)** 0 **(C)** 1 **(D)** $\pi/2$ **(E)** cannot be uniquely determined

12. Suppose for every positive x that

$$xe^x = e + \int_1^{x^3} f(t)\,dt$$

Find the value of $f(8)$.

(A) $e/4$ **(B)** e^2 **(C)** $e^2/4$ **(D)** $3e^2$ **(E)** 6

13. Find the area inside the ellipse given by

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

(A) 5π **(B)** 6π **(C)** $25\pi/4$ **(D)** 7π **(E)** 8π

14. Let $\sum_{n=1}^{\infty} a_n$ be a positive term convergent series. Which of the following series must converge?

$$\text{I.} \quad \sum_{n=1}^{\infty} \frac{1}{a_n}$$

$$\text{II.} \quad \sum_{n=1}^{\infty} \sqrt{a_n}$$

$$\text{III.} \quad \sum_{n=1}^{\infty} a_n^2$$

(A) I only **(B)** II only **(C)** III only **(D)** II and III only **(E)** I, II, and III

15. Let $I_n = \int_0^1 x^n e^{-x}\, dx$, where n is a positive integer. Which of the following is true?

$$\begin{array}{ll} \text{I.} & I_n = -e^{-1} + nI_{n-1} \quad \text{for } n \geq 2 \\ \text{II.} & I_n = n! - [en!]e^{-1} \quad \text{for } n \geq 2 \\ \text{III.} & \lim\limits_{n\to\infty} I_n = 0 \end{array}$$

where the notation $[x]$ stands for the greatest integer less than or equal to x.

(A) I only **(B)** II only **(C)** III only **(D)** I and III only **(E)** I, II, and III

16. What is the coefficient of x^{10} in the expansion of

$$(1+x)(1+x^2)(1+x^3)\cdots(1+x^{10})$$

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 32

17. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

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(A) 10 **(B)** 7 **(C)** 0 **(D)** -3 **(E)** -7

19. Hezy tosses a fair cubical die until each number 1 through 6 has come up at least once. On average, Hezy requires X tosses to do this. The real number X is closest to

(A) 12.8 **(B)** 14.7 **(C)** 16.3 **(D)** 17.2 **(E)** 19.5

20. In $\triangle ABC$, points D, E, and F are located on $\overline{BC}$, $\overline{AC}$, and $\overline{BA}$ respectively, so that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are concurrent at point P, the area of $\triangle BPD$ is 190, the area of $\triangle DPC$ is 380, and the area of $\triangle CPE$ is 418. Then the area of $\triangle APE$ is

(A) 121 **(B)** 143 **(C)** 242 **(D)** 319 **(E)** 330

Spring 2010 McNabb GDCTM Contest
Level J1

1. How many diagonals does a hexagon have?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2. Amy is in a line of 40 people at the movies. Ahead of her stand 12 people. How many stand behind her?

   (A) 13 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 39

3. What is the smallest positive integer which has the five smallest primes as factors?

   (A) 209 **(B)** 210 **(C)** 2310 **(D)** 15015 **(E)** 100000

4. At the post office Amy spent a total of \$ 5.00 to buy some 43 cent stamps and some 5 cent stamps. How many 5 cent stamps could she have bought?

   (A) 7 **(B)** 10 **(C)** 11 **(D)** 13 **(E)** 14

5. The average of two positive numbers is equal to twice the smaller of the two numbers. How many times greater is the larger number than the smaller?

   (A) 1.5 times **(B)** 2 times **(C)** 2.5 times **(D)** 3 times **(E)** 4 times

6. A kangaroo is 200 feet from a rabbit, when the kangaroo starts chasing the rabbit. Both immediately start hopping in the same direction. For each 13 foot leap of the kangaroo the rabbit takes two 4 foot leaps. From the time the chase began until the rabbit is caught, how many leaps did the rabbit take?

   (A) 20 **(B)** 40 **(C)** 80 **(D)** 100 **(E)** 200

7. When three distinct numbers from the set $\{9, 8, -2, -4, -5\}$ are multiplied, the largest possible product is

   (A) 64 **(B)** 90 **(C)** 160 **(D)** 180 **(E)** 360

8. The Elm school girls' basketball team has 11 girls on the team. The team will play 22 games this season, with each game lasting 32 minutes. The coach arranges for each girl to have the same total playing time by the end of the season. How many total minutes playing time would each girl end up with at the end of the season?

   (A) 64 **(B)** 128 **(C)** 200 **(D)** 320 **(E)** 440

9. Three pomegranates and one pineapple weigh as much as sixteen plums. Four plums and one pomegranate weight as much as one pineapple. How many pomegranates weigh as much as 3 pineapples?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

10. A box contains 10 red marbles, 11 blue marbles, and 12 green marbles. What is the fewest number of marbles you must pull out of the box to be sure of getting at least 5 of the same color?

(A) 5 **(B)** 10 **(C)** 13 **(D)** 26 **(E)** 28

11. What is the area of the quadrilateral in the coordinate plane with vertices whose coordinates are (in order): $(0,0)$, $(7,1)$, $(4,4)$, and $(2,11)$?

(A) 30 **(B)** 31 **(C)** 31.5 **(D)** 32 **(E)** 33

12. Ronald has an unlimited number of 5 cent and 7 cent stamps. What is the largest amount of postage (in cents) that he cannot make with these stamps?

(A) 16 **(B)** 22 **(C)** 23 **(D)** 79 **(E)** 99

13. How many elements are in the set $\{7, 11, 15, 19, \cdots, 403\}$?

(A) 99 **(B)** 100 **(C)** 101 **(D)** 102 **(E)** 397

14. Abigail, Brice, and Carl all start out with some one-dollar bills in their pockets. In particular, Carl starts with 4 one-dollar bills. Brice then gives half of his dollar bills to Abigail and the other half to Carl. Then Abigail gives half of her bills to Brice and the other half to Carl. Finally, Carl gives half of his bills to Abigail and keeps the rest. If now, at the end of these exchanges, Abigail, Brice, and Carl all have 8 one-dollar bills, how many such bills did Abigail have to begin with?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

15. Farmer Ben sells thimbleberry jam in cylindrical jars. The company that supplies his jars is discontinuing the size jar he is currently using. Farmer Ben will have to order jars that have a diameter 10% less than his current jar. To maintain the volume of his current jar, the new jars he orders should have a height what percent greater than the current ones? Answer to the nearest percent.

(A) 11 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 124

16. Let a, b, c, and d be positive real numbers such that $abc = 12$, $bcd = 6$, $acd = 125$, and $abd = 3$. Then $abcd$ equals

(A) 12 **(B)** 15 **(C)** 30 **(D)** 45 **(E)** 60

17. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

(A) $\frac{5}{72}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{16}$ **(E)** $\frac{3}{8}$

18. If a is a multiple of 14 and b is a multiple of 21, then what is the largest integer that must be a factor of any integer of the form $9a + 8b$?

(A) 84 **(B)** 42 **(C)** 21 **(D)** 14 **(E)** 8

19. Each vertex of a cube is randomly colored red or blue with each color being equally likely. What is the probability that every pair of adjacent vertices have different colors?

(A) 0 **(B)** 1/128 **(C)** 1/64 **(D)** 1/32 **(E)** 1/2

20. The value of the expression

$$(1 - (2 - (3 - (4 - (5 - (\cdots - (n)))) \cdots)$$

for n a positive even integer is equal to

(A) $-n$ **(B)** $-n/2$ **(C)** 0 **(D)** $n/2$ **(E)** $n - 3$

21. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1 inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(A) 576 **(B)** 864 **(C)** 1152 **(D)** 1200 **(E)** 1600

22. The sequence $1, 3, 6, 10, 15, 21, \cdots$ is called the sequence of *triangular* numbers, because those numbers of dots can be arranged to form equilateral triangles. For instance, the triangular number 10 occurs in the set-up of bowling pins. What is the smallest triangular number greater than 1000?

(A) 2^{10} **(B)** 1035 **(C)** 1021 **(D)** 1010 **(E)** 1006

23. In how many ways can five distinct books be arranged in a bookcase with 3 shelves, each shelf capable of holding all five books?

(A) 19 **(B)** 120 **(C)** 360 **(D)** 840 **(E)** 2520

24. The probability that Gerald wins any given game of HORSE is 3/5. Next Saturday, Gerald will play exactly five games of HORSE. What is the probability that he will win exactly three of them?

(A) $\frac{108}{3125}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{216}{625}$ **(D)** $\frac{9}{25}$ **(E)** 1

25. Seven stacks are made each consisting of seven half-dollar coins. One entire stack is made of counterfeit coins. All other stacks have true half-dollars. You know the weight of true half-dollars in grams. And each counterfeit half-dollar weighs exactly one gram more than the true coin. You can weigh the coins or any subset of them on a digital scale (similar to a regular bathroom scale) which outputs in grams. What is the minimum number of weighings needed to determine which stack is the counterfeit one?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 7

Spring 2010 McNabb GDCTM Contest
Level J2

1. Amy is in a line of 40 people at the movies. Ahead of her stand 12 people. How many stand behind her?

   (A) 13 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 39

2. Simplify $a + a + a + a + a + a + a + a$.

   (A) $4a$ **(B)** $5a$ **(C)** $6a$ **(D)** $7a$ **(E)** $8a$

3. If the least common multiple of a and b is 38, what is the least common multiple of $15a$ and $15b$?

   (A) 38 **(B)** 114 **(C)** 190 **(D)** 570 **(E)** not uniquely determined

4. Three pomegranates and one pineapple weigh as much as sixteen plums. Four plums and one pomegranate weight as much as one pineapple. How many pomegranates weigh as much as 3 pineapples?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

5. What is the area of the quadrilateral in the coordinate plane with vertices whose coordinates are (in order): $(0, 0)$, $(7, 1)$, $(4, 4)$, and $(2, 11)$?

   (A) 30 **(B)** 31 **(C)** 31.5 **(D)** 32 **(E)** 33

6. Ronald has an unlimited number of 5 cent and 7 cent stamps. What is the largest amount of postage (in cents) that he cannot make with these stamps?

   (A) 16 **(B)** 22 **(C)** 23 **(D)** 79 **(E)** 99

7. If r is a solution of the equation $x^2 + 11x + 19 = 0$, what is the value of $(r + 5)(r + 6)$?

   (A) -15 **(B)** -11 **(C)** 0 **(D)** 7 **(E)** 11

8. Abigail, Brice, and Carl all start out with some one-dollar bills in their pockets. In particular, Carl starts with 4 one-dollar bills. Brice then gives half of his dollar bills to Abigail and the other half to Carl. Then Abigail gives half of her bills to Brice and the other half to Carl. Finally, Carl gives half of his bills to Abigail and keeps the rest. If now, at the end of these exchanges, Abigail, Brice, and Carl all have 8 one-dollar bills, how many such bills did Abigail have to begin with?

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

9. If $f(x)$ is a linear function for which $f(8) - f(1) = 11$, then $f(41) - f(6)$ is equal to

   (A) 61 **(B)** 55 **(C)** 49 **(D)** 43 **(E)** 37

10. Let a, b, c, and d be positive real numbers such that $abc = 12$, $bcd = 6$, $acd = 125$, and $abd = 3$. Then $abcd$ equals

(A) 12 **(B)** 15 **(C)** 30 **(D)** 45 **(E)** 60

11. The surface area of a large spherical balloon is doubled. By what factor is the volume of the balloon increased?

(A) 8 **(B)** 4 **(C)** $2\sqrt{2}$ **(D)** $\sqrt[3]{4}$ **(E)** 2

12. Zeke cycles steadily for 36 miles. If he had managed to go 3 mph faster, he would have taken one hour less for the trip. What was Zeke's actual speed in mph during this trip?

(A) 9 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 21

13. A set of seven distinct positive integers has a mean of 13. Find the difference between the greatest possible median of these integers and the least possible median of these integers.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

14. A line L in the coordinate plane has slope -2. Suppose the triangle with vertices given by the origin, the x-intercept of L, and the y-intercept of L has area 9. Then an equation for L could be

(A) $2x + y = 0$ **(B)** $2x + y = 4$ **(C)** $-2x + y = 6$
(D) $2x + y = 3$ **(E)** $2x + y = -6$

15. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

(A) $\frac{5}{72}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{16}$ **(E)** $\frac{3}{8}$

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17. Each vertex of a cube is randomly colored red or blue with each color being equally likely. What is the probability that every pair of adjacent vertices have different colors?

(A) 0 **(B)** 1/128 **(C)** 1/64 **(D)** 1/32 **(E)** 1/2

18. One root of $2x^2 + 15x + c$ is four times the other. What is the value of c?

(A) 36 **(B)** 9 **(C)** 18 **(D)** 3/2 **(E)** -3/2

19. The product of three distinct positive integers is 210. What is the maximum possible sum of these three integers?

(A) 18 **(B)** 38 **(C)** 74 **(D)** 108 **(E)** 212

20. The value of the expression

$$(1-(2-(3-(4-(5-(\cdots-(n))))\cdots)$$

for n a positive even integer is equal to

(A) $-n$ **(B)** $-n/2$ **(C)** 0 **(D)** $n/2$ **(E)** $n-3$

21. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1 inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(A) 576 **(B)** 864 **(C)** 1152 **(D)** 1200 **(E)** 1600

22. Two ferries start at the same instant from opposite banks of a river. They travel directly across the river. Each boat keeps its own constant speed, though one boat is faster than the other. In this first trip across they pass at a point 720 yards from the nearer bank. When reaching the opposite shore each boat remains exactly 10 minutes in its dock before heading back the other way. On this trip back the boats meet 400 yards from the other shore. How wide is the river (in yards)?

(A) 1040 **(B)** 1120 **(C)** 1520 **(D)** 1600 **(E)** 1760

23. In how many ways can five distinct books be arranged in a bookcase with 3 shelves, each shelf capable of holding all five books?

(A) 19 **(B)** 120 **(C)** 360 **(D)** 840 **(E)** 2520

24. The probability that Gerald wins any given game of HORSE is 3/5. Next Saturday, Gerald will play exactly five games of HORSE. What is the probability that he will win exactly three of them?

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(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 7