Spring 2018 McNabb GDCTM Contest
Calculus

NO Calculators Allowed/ 60 Minutes

Assume all variables are real unless otherwise stated in the problem.

1. A tennis tournament has ten players registered to play singles. How many different first-round pairings are possible? In the first-round there are five matches and all ten players play.

2. Find the sum of the coefficients of all the even powers of x when

$$(x^5 - x^2 + 1)^{10}$$

is expanded and simplified.

3. In how many ways can you choose five of these letters

$$a, b, c, d, e, f, g, h, i, j, k, x, x, x, x, x$$

?

4. Find $\dfrac{d^{10}y}{dx^{10}}$ if $y = e^{-x}\cos x$.

5. For which values of the parameter a are all the roots of the polynomial $x^4 + ax^2 + 1$ real?

6. Find the absolute maximum value of the function $f(\theta) = \cos^3\theta \sin\theta$ over the interval $[0, \pi/2]$.

7. There exists a function continuous at each real number but differentiable at no real number. Answer True or False.

8. For what values of the real parameter a is the function

$$f(x) = x^4 + ax^3 + ax^2 + ax + a$$

concave up on the entire real number line? Answer in interval notation.

9. Find

$$\lim_{x\to\infty} \sqrt{x + \sqrt{x}} - \sqrt{x}$$

10. Find the smallest possible value of the the constant m such that the inequality $mx - 1 + 1/x \geq 0$ holds for all $x > 0$.

11. Find the maximum value of the expression

$$16x^5 - 20x^3 + 5x$$

as x varies over the interval $-1 \leq x \leq 1$.

12. A certain radioactive isotope has a mean time to decay of 12 seconds. Given a very large number of such atoms, how many seconds do you have to wait until half of them have decayed?

13. Find the first four non-zero terms of the Taylor polynomial approximation centered at zero of the solution of the differential equation

$$\frac{dy}{dx} = xy + 1$$

with initial condition $y(0) = 3$.

14. Find a function $f(t)$ that satisfies for all x

$$16x^2 + 16x + 4 = \int_{3x+1}^{5x+2} f(t)\,dt$$

15. Find the maximum possible value of $2a + b$ if $a \geq 0$, $b \geq 0$, and $8a^3 = 8ab - b^3$.