NO Calculators Allowed

Assume all variables are real unless otherwise stated in the problem.

1. What is the largest prime p satisfying $\left(\dfrac{16}{p}\right)^2 > 2$?

2. Find the value of x if $4^5 + 4^5 + 4^5 = 2^x + 2^x + 2^x$.

3. Find the integer m such that
$$(\sqrt{2} - 1)^4 = \sqrt{m} - \sqrt{m-1}$$

4. Find the value of n if $\log_{48} 12 = \log_n 144$.

5. In how many different ways can 10 identical chairs be distributed to 4 distinct tables? A table may be left without any chairs at all. Answer in standard integer form.

6. In which quadrant (I, II, III, or IV) of the complex plane does $(-\sqrt{3} + 3i)^{17}$ lie?

7. Suppose that for all $1 < x < 3$

$$|7x + 4 - f(x)| \le (|x - 2|)^{3/2}$$

   Find the value of $f'(2)$.

8. Find all values of the parameter r so that $y(x) = e^{rx}$ satisfies $y''(x) + 5y'(x) = 14y(x)$ for all x.

9. Let $f(x) = x^4 + ax^3 + (a+2)x^2$. On what maximal open interval or union of open intervals of the parameter a is $f(x)$ concave up for all x.?

10. If f is a differentiable function on $[-3,3]$ satisfying $f(1) = -1$, $f(-1) = 1$, $f'(1) = 2$ and $f'(-1) = -2$, find the value of $(f \circ f \circ f \circ f)'(1)$.

11. Find the largest value of the constant c so that

$$x^4 + 9 \ge cx$$

   holds for all x.

12. Let g be twice-differentiable on the interval $[0,5]$. Let $g(0) = g'(0) = 0$ and suppose $g''(x) \le 6$ for all x in $[0,5]$. What is the maximum possible value of $g(4)$?

13. The set of all tangent lines to a function $f(x)$ can be described as

$$\{y = a(2x - a) + 2 - 4x \ : \ a \in \mathbb{R}\}$$

   Find an algebraic formula for this function $f(x)$ in terms of x.

14. Find $g'''(0)$ if
$$g(x) = \frac{x+1}{1 - x^3 - x^4}$$

15. Determine the value of
$$\lim_{x \to 0} \frac{\ln(1 + 2x + x^2) + \ln(1 - 2x + x^2)}{x(e^x - 1)}$$