

SPRING 2013 McNABB GDCTM CONTEST
CALCULUS

NO Calculators Allowed

Assume all variables are real unless otherwise stated in the problem.

1. How many positive factors does 2013 have?

- (A) 6 (B) 8 (C) 10 (D) 12 (E) 14

2. The value of

$$1 + 2 + 3 + 4 - 5 + 6 + 7 + 8 + 9 - 10 + \cdots + 46 + 47 + 48 + 49 - 50$$

is equal to

- (A) 600 (B) 650 (C) 725 (D) 750 (E) 800

3. I have two numbers in mind. The first number leaves a remainder of 4159 when divided by 5153 while the second number leaves a remainder of 5149 when divided by 5153. What is the remainder when the sum of these numbers is divided by 5153?

- (A) 3135 (B) 3455 (C) 4144 (D) 4155 (E) 4344

4. If the equations $x^2 + ax + 21 = 0$ and $2x^2 + 19x + 35 = 0$ have a solution in common, what could be the value of the constant a ?

- (A) -10 (B) -4 (C) -2 (D) 4 (E) 10

5. Which transformation never changes the median of a list of a dozen distinct positive integers?

- (A) adding 6 to each number in the list
(B) adding 3 to each of the three smallest numbers in the list
(C) subtracting 4 from each of the four largest numbers in the list
(D) doubling each number in the list
(E) taking the reciprocal of each number in the list

6. Which of these numbers is the least?

- (A) $\log_8 144$ (B) $\log_4 72$ (C) $\log_{16} 288$ (D) $\log_2 48$ (E) $\log_{32} 576$

7. A careless librarian has reshelfed the 5 volumes of an art encyclopedia in the correct order. Each volume has its spine facing out, which is correct of course, but has a $1/4$ probability of being upside down. What is the probability that exactly one pair of front covers are now face to face?

- (A) $1/64$ (B) $2/31$ (C) $3/16$ (D) $5/24$ (E) $69/128$

8. Recall that $i = \sqrt{-1}$. What is the sum of the infinite geometric series $\sum_{n=0}^{\infty} (i/2)^n$?

- (A) $-\frac{1}{5} + \frac{2}{5}i$ (B) $\frac{3}{5} - \frac{1}{5}i$ (C) $\frac{4}{5} + \frac{2}{5}i$ (D) 0 (E) i

9. The set of points in space equidistant from two skew lines is

- (A) the empty set (B) a single point (C) a line
(D) the union of two intersecting lines (E) none of the above

10. How many solutions in radians of $\sin 2\theta = \cos 3\theta$ lie in the interval $[0, 2\pi]$?

- (A) 0 (B) 2 (C) 3 (D) 4 (E) 6

11. The integral

$$\int_0^{\pi/2} \frac{1}{1 + \cos \theta} d\theta$$

has value

- (A) $3/5$ (B) $5/6$ (C) 1 (D) $7/5$ (E) diverges

12. Find the minimum possible value of the expression $6 \cos x + 2 \cos 2x + 5$.

- (A) $2/3$ (B) $3/4$ (C) $4/5$ (D) $5/6$ (E) 1

13. A thin rod lies along the x -axis with endpoints at $x = 2$ and $x = 8$. If the density of the rod at each point is directly proportional to the point's distance to the origin, what is the x -coordinate of the center of mass of the rod?

(A) $19/5$ (B) 4 (C) $14/3$ (D) $28/5$ (E) 5

14. How many values of the constant k satisfy both: (i) $k \geq 1$ and (ii) $\int_1^k (2k - 2)x^k dx = 80$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

15. Determine

$$\lim_{n \rightarrow \infty} \int_0^{\pi/6} (\sin x)^n dx$$

(A) 0 (B) $1/10$ (C) $\pi/12$ (D) $1/2$ (E) does not exist

16. The improper integral $\int_0^\infty \frac{1}{1 + e^x} dx$ has the value

(A) $\ln 2$ (B) $1/2$ (C) $2/3$ (D) e (E) does not converge

17. Given that $\int_0^{10} \ln(x^2 - 10x + 26) dx = k$ then find the value of

$$\int_0^{10} x \ln(x^2 - 10x + 26) dx$$

(A) 0 (B) k (C) $2k$ (D) $k \ln 2$ (E) $5k$

18. The coefficient of x^8 in the Maclaurin power series of $f(x) = \frac{1 + 2x}{1 - x - x^2}$ is equal to

(A) 47 (B) 76 (C) 91 (D) 101 (E) 123