Spring 2010 McNabb GDCTM Contest
Level J2

1. Amy is in a line of 40 people at the movies. Ahead of her stand 12 people. How many stand behind her?

   (A) 13 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 39

2. Simplify $a+a+a+a+a+a+a+a$.

   (A) $4a$ **(B)** $5a$ **(C)** $6a$ **(D)** $7a$ **(E)** $8a$

3. If the least common multiple of a and b is 38, what is the least common multiple of $15a$ and $15b$?

   (A) 38 **(B)** 114 **(C)** 190 **(D)** 570 **(E)** not uniquely determined

4. Three pomegranates and one pineapple weigh as much as sixteen plums. Four plums and one pomegranate weight as much as one pineapple. How many pomegranates weigh as much as 3 pineapples?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

5. What is the area of the quadrilateral in the coordinate plane with vertices whose coordinates are (in order): $(0,0)$, $(7,1)$, $(4,4)$, and $(2,11)$?

   (A) 30 **(B)** 31 **(C)** 31.5 **(D)** 32 **(E)** 33

6. Ronald has an unlimited number of 5 cent and 7 cent stamps. What is the largest amount of postage (in cents) that he cannot make with these stamps?

   (A) 16 **(B)** 22 **(C)** 23 **(D)** 79 **(E)** 99

7. If r is a solution of the equation $x^2+11x+19=0$, what is the value of $(r+5)(r+6)$?

   (A) -15 **(B)** -11 **(C)** 0 **(D)** 7 **(E)** 11

8. Abigail, Brice, and Carl all start out with some one-dollar bills in their pockets. In particular, Carl starts with 4 one-dollar bills. Brice then gives half of his dollar bills to Abigail and the other half to Carl. Then Abigail gives half of her bills to Brice and the other half to Carl. Finally, Carl gives half of his bills to Abigail and keeps the rest. If now, at the end of these exchanges, Abigail, Brice, and Carl all have 8 one-dollar bills, how many such bills did Abigail have to begin with?

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

9. If $f(x)$ is a linear function for which $f(8)-f(1)=11$, then $f(41)-f(6)$ is equal to

   (A) 61 **(B)** 55 **(C)** 49 **(D)** 43 **(E)** 37

10. Let a, b, c, and d be positive real numbers such that $abc = 12$, $bcd = 6$, $acd = 125$, and $abd = 3$. Then $abcd$ equals

(A) 12 **(B)** 15 **(C)** 30 **(D)** 45 **(E)** 60

11. The surface area of a large spherical balloon is doubled. By what factor is the volume of the balloon increased?

(A) 8 **(B)** 4 **(C)** $2\sqrt{2}$ **(D)** $\sqrt[3]{4}$ **(E)** 2

12. Zeke cycles steadily for 36 miles. If he had managed to go 3 mph faster, he would have taken one hour less for the trip. What was Zeke's actual speed in mph during this trip?

(A) 9 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 21

13. A set of seven distinct positive integers has a mean of 13. Find the difference between the greatest possible median of these integers and the least possible median of these integers.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

14. A line L in the coordinate plane has slope -2. Suppose the triangle with vertices given by the origin, the x-intercept of L, and the y-intercept of L has area 9. Then an equation for L could be

(A) $2x + y = 0$ **(B)** $2x + y = 4$ **(C)** $-2x + y = 6$
(D) $2x + y = 3$ **(E)** $2x + y = -6$

15. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

(A) $\frac{5}{72}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{16}$ **(E)** $\frac{3}{8}$

16. If a is a multiple of 14 and b is a multiple of 21, then what is the largest integer that must be a factor of any integer of the form $9a + 8b$?

(A) 84 **(B)** 42 **(C)** 21 **(D)** 14 **(E)** 8

17. Each vertex of a cube is randomly colored red or blue with each color being equally likely. What is the probability that every pair of adjacent vertices have different colors?

(A) 0 **(B)** 1/128 **(C)** 1/64 **(D)** 1/32 **(E)** 1/2

18. One root of $2x^2 + 15x + c$ is four times the other. What is the value of c?

(A) 36 **(B)** 9 **(C)** 18 **(D)** 3/2 **(E)** -3/2

19. The product of three distinct positive integers is 210. What is the maximum possible sum of these three integers?

(A) 18 **(B)** 38 **(C)** 74 **(D)** 108 **(E)** 212

20. The value of the expression

$$(1-(2-(3-(4-(5-(\cdots-(n))))\cdots)$$

for n a positive even integer is equal to

(A) $-n$ **(B)** $-n/2$ **(C)** 0 **(D)** $n/2$ **(E)** $n-3$

21. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1 inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(A) 576 **(B)** 864 **(C)** 1152 **(D)** 1200 **(E)** 1600

22. Two ferries start at the same instant from opposite banks of a river. They travel directly across the river. Each boat keeps its own constant speed, though one boat is faster than the other. In this first trip across they pass at a point 720 yards from the nearer bank. When reaching the opposite shore each boat remains exactly 10 minutes in its dock before heading back the other way. On this trip back the boats meet 400 yards from the other shore. How wide is the river (in yards)?

(A) 1040 **(B)** 1120 **(C)** 1520 **(D)** 1600 **(E)** 1760

23. In how many ways can five distinct books be arranged in a bookcase with 3 shelves, each shelf capable of holding all five books?

(A) 19 **(B)** 120 **(C)** 360 **(D)** 840 **(E)** 2520

24. The probability that Gerald wins any given game of HORSE is 3/5. Next Saturday, Gerald will play exactly five games of HORSE. What is the probability that he will win exactly three of them?

(A) $\dfrac{108}{3125}$ **(B)** $\dfrac{3}{5}$ **(C)** $\dfrac{216}{625}$ **(D)** $\dfrac{9}{25}$ **(E)** 1

25. Seven stacks are made each consisting of seven half-dollar coins. One entire stack is made of counterfeit coins. All other stacks have true half-dollars. You know the weight of true half-dollars in grams. And each counterfeit half-dollar weighs exactly one gram more than the true coin. You can weigh the coins or any subset of them on a digital scale (similar to a regular bathroom scale) which outputs in grams. What is the minimum number of weighings needed to determine which stack is the counterfeit one?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 7