Fall 2014 McNabb GDCTM Contest
PreCalculus

NO Calculators Allowed

1. Express 188 as the sum of two prime numbers.

2. Jack and Jill start walking toward each other. Initially they were 700 meters apart. Jack walks 4/3 as fast as Jill. When they meet, how far is Jack from where Jill started?

3. Lincoln Middle School has 324 students and 17 teachers. A field trip for the entire school (students and teachers) to the Kimball Museum is planned. Each of their buses holds at most 28 passengers. Each bus must have at least one teacher on it. What is the minimum number of buses the school requires for this field trip?

4. If a ounces of tea leaves brews b cups of tea and c cups fill one thermos, how many ounces of tea leaves must be brewed to fill d thermos's? Answer in terms of a, b, c, and d.

5. The pages of the book *Science of Mechanics in the Middle Ages* are numbered from 1 to 711. Considering all the digits needed to print these page numbers starting from page 1, on what page number does the 241st '1'occur?

6. Recall that $\binom{m}{n}$ stands for the number of ways of choosing n objects from a set of m objects. Name a solution m greater than 2 of the equation

$$\binom{m+2}{3} = 4\binom{m}{2}$$

7. Find the value of

$$\sum_{k=1}^{100} i^{k(k+1)/2}$$

   Here, i stands for the square root of negative one.

8. Suppose n is a positive integer greater than or equal to 10. For such n, define $f(n)$ to be sum of the units digit of n and twice the value of $f(m)$ where m is the integer that remains when the units digit of n is removed. In case n is a positive integer less than or equal to 9, define $f(n) = n$. Find the value of $f(1145)$.

9. Let $\lfloor x \rfloor$ be the greatest integer which is less than or equal to x. Write the solution of the equation

$$\lfloor 2x \rfloor = \lfloor 3x \rfloor$$

   using interval notation.

10. Define recursively the function $a(m, n)$:

$$a(m, n) = a(m - 1, a(m, n - 1))$$
$$a(0, n) = n + 1$$
$$a(m, 0) = a(m - 1, 1)$$

   Find the value of $a(2, 2)$.

11. Find the volume of the tetrahedron with vertices located at $(0,0,0)$, $(1,-2,1)$, $(1,2,-1)$, and $(2,1,-1)$.

12. The line $y = mx$ intersects the lines $x + y = 7$ and $x + y = -14$ at points A and B respectively. If $AB = 39$, what is a possible value for the slope m?

13. In $\triangle ABC$ point D lies on side BC so that AD bisects angle BAC. If $AD = 13$, $DC = 37$, and $AC = 40$, find the length of AB.

14. Let $f(x)$ be a function which satisfies for all x and y the relation

$$f(x) \cdot f(y) - f(xy) = x + y$$

Determine the function $f(x)$.

15. In how many ways can 4 different rings be placed on the four fingers of the right hand? Here the order of the rings on a given finger matters and each finger can accomodate all four rings.