Fall 2013 McNabb GDCTM Contest
PreCalculus

NO Calculators Allowed

1. How many six-digit integers of the form $79A4B1$, where A and B are digits, are divisible by eleven?

2. A set S of ordered pairs is said to be *transitive* if whenever (a,b) and (b,c) belong to S then so does (a,c). Is this set S below transitive?

$$S = \{(6,13),(4,8),(5,7),(6,10),(3,5),(10,13),(3,7),(1,5),\\(3,10),(1,4),(1,7),(9,6)\}$$

   Answer Yes or No.

3. Find the value of x if $4^5 + 4^5 + 4^5 = 2^x + 2^x + 2^x$.

4. For which integer m does $\frac{m}{13} < \sqrt{2} < \frac{m+1}{13}$ hold?

5. In how many different ways can 10 identical chairs be distributed to 4 distinct tables? A table may be left without any chairs at all. Answer in standard integer form.

6. Determine how many ordered pairs of positive integers (a,b) satisfy $\frac{1}{a} + \frac{2}{b} = \frac{3}{7}$.

7. Find the set of all values of the parameter a so that the graph of the parabola $y = ax^2 + 2x + 4a$ never enters the third quadrant III. Recall that $III = \{(x,y) : x < 0 \text{ and } y < 0\}$.

8. Point A has coordinates $(19, -104/3)$ while point B has coordinates $(-43/3, -14)$. Find the coordinates of a point C which lies on the perpendicular bisector of segment AB given that both of the coordinates of C are integers.

9. Find the largest value of c for which the following inequality holds for all x:

$$x^4 + 7x^2 + 144 \geq cx^2$$

10. Find the value of n if $\log_{48} 12 = \log_n 144$.

11. Find all real solutions of the equation

$$0 = (2x+1)^5 + (2x+1)^4(2x-1) + (2x+1)^3(2x-1)^2\\ + (2x+1)^2(2x-1)^3 + (2x+1)(2x-1)^4 + (2x-1)^5$$

12. In a triangle one of the angles is twice another and the sides opposite these angles have lengths 16 and 28. Find the length of the third side of this triangle.

13. Find the number of ordered pairs of integers (m,n) that satisfy

$$m^2 - mn + n^2 - m - n = 0$$

14. Find the maximum possible product of a set of positive integers whose sum is 27. Answer in standard integer form.

15. Let z be a complex number. If $z = a + bi$ where a and b are real numbers, find how many such z with $b < 0$ satisfy

$$z^7 + z^5 + z^3 + z = 9$$