Spring 2013 McNabb GDCTM Contest
PreCalculus

NO Calculators Allowed

1. A rectangle with area 125 has its sides in the ratio of $4 : 5$. What is the perimeter of this rectangle?

   (A) 18 **(B)** 22.5 **(C)** 36 **(D)** 45 **(E)** 54

2. In the repeating decimal $0.\overline{71771}$, in which decimal place does the 2013th 7 appear?

   (A) 671st **(B)** 2014th **(C)** 2015th **(D)** 3354th **(E)** 3355th

3. How many seconds are there in exactly six weeks?

   (A) 7! **(B)** 8! **(C)** 9! **(D)** 10! **(E)** 12!

4. The sum of a set of numbers is the sum of all the numbers in that set. How many subsets of the set $\{1, 2, 3, 4, 5, 6, 7\}$ have a sum of 12?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

5. The value of

   $$1 + 2 + 3 + 4 - 5 + 6 + 7 + 8 + 9 - 10 + \cdots + 46 + 47 + 48 + 49 - 50$$

   is equal to

   (A) 600 **(B)** 650 **(C)** 725 **(D)** 750 **(E)** 800

6. A solid opaque cube of side length 5 meters rests on flat ground. It is illuminated only by a powerful point-source of light located 5 meters above one of the cube's top corners. Find the area in square meters of the shadow cast by the cube on the ground.

   (A) 75 **(B)** 85 **(C)** $50\sqrt{2}$ **(D)** 91 **(E)** 100

7. Let a, b, and n be constants, with n a positive integer. If the first three terms of the binomial expansion of $(a+x)^n$ are, in ascending powers of x, equal to $3b + 6bx + 5bx^2$, then find the value of $a + b + n$.

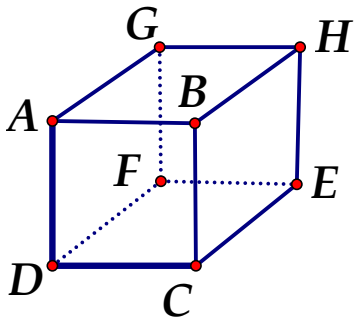
(A) 48 **(B)** 64 **(C)** 96 **(D)** 128 **(E)** 252

8. Which of these numbers is the least?

(A) $\log_8 144$ **(B)** $\log_4 72$ **(C)** $\log_{16} 288$ **(D)** $\log_2 48$ **(E)** $\log_{32} 576$

9. In cube $ABCDEFGH$ shown find $\cot \angle DBF$

(A) $2/\sqrt{6}$ **(B)** $5/6$ **(C)** 1 **(D)** $\sqrt{2}$ **(E)** $6/5$



10. Which of the following equations has exactly two solutions over the real numbers?

(A) $x^2 - 6x + 9 = 0$ **(B)** $5x = 2(5 - 7x)$ **(C)** $|x + 8| = -5$
(D) $|x| = 12$ **(E)** $x^2 + 1 = 0$

11. How many solutions in radians of $\sin 2\theta = \cos 3\theta$ lie in the interval $[0, 2\pi]$?

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

12. Recall that $i = \sqrt{-1}$. What is the sum of the infinite geometric series $\sum_{n=0}^{\infty} (i/2)^n$?

(A) $-\frac{1}{5} + \frac{2}{5}i$ **(B)** $\frac{3}{5} - \frac{1}{5}i$ **(C)** $\frac{4}{5} + \frac{2}{5}i$ **(D)** 0 **(E)** i

13. Given the three points $(2013, -1863)$, $(1776, -1812)$, and $(1181, -1492)$ in the coordinate plane, a fourth point (a, b) is called a *complementing* point if it along with the given three points form the vertices of a parallelogram. Find the sum of all the coordinates of all the complementing points of the given three points.

(A) -197 **(B)** 0 **(C)** 216 **(D)** 631 **(E)** 783

14. When $x^{101} + x^{51} + 1$ is divided by $x^3 + 1$, what is the remainder?

(A) 0 **(B)** x **(C)** $3x^2 + 4x - 2$ **(D)** -1 **(E)** $-x^2$

15. Let $f(x) = (1/4)x^2 + bx + c$ where b and c are constants. If b and c are chosen randomly and independently from the set of digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ what is the probability that the vertex of the parabola $y = f(x)$ lies on the x-axis?

(A) $1/25$ **(B)** $1/20$ **(C)** $1/10$ **(D)** $4/25$ **(E)** $1/5$

16. A careless librarian has reshelved the 5 volumes of an art encyclopedia in the correct order. Each volume has its spine facing out, which is correct of course, but has a $1/4$ probability of being upside down. What is the probability that exactly one pair of front covers are now face to face?

(A) $1/64$ **(B)** $2/31$ **(C)** $3/16$ **(D)** $5/24$ **(E)** $69/128$

17. The set of points in space equidistant from two skew lines is

(A) the empty set **(B)** a single point **(C)** a line
(D) the union of two intersecting lines **(E)** none of the above

18. In triangle ABC, the angle bisector CD of $\angle C$ has point D on side AB. If $AC = 1$, $BC = \sqrt{3}$, $AD = \sqrt{3} - 1$ and $DB = 3 - \sqrt{3}$, then what is the length CD?

(A) $\sqrt{1 + \sqrt{3}}$ **(B)** $\sqrt{6 - 3\sqrt{3}}$ **(C)** $9/10$ **(D)** 1 **(E)** $1/\sqrt{2}$