Spring 2012 McNabb GDCTM Contest
Pre-calculus

NO Calculators Allowed

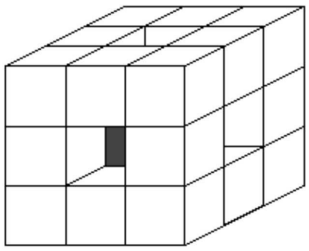
Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. The value of $-3-3^2-3^3$ is equal to

   (A) -41 **(B)** -40 **(C)** -39 **(D)** -37 **(E)** -36

2. Twenty seven small $1\times1\times1$ cubes are glued together to form a $3\times3\times3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

   (A) 56 **(B)** 64 **(C)** 72 **(D)** 84 **(E)** 96



3. What is the remainder when the sum

$$1^{111}+2^{111}+3^{111}+4^{111}+5^{111}+6^{111}+7^{111}+8^{111}+9^{111}+10^{111}$$

   is divided by 11?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

4. In triangle ABC, put $AB=c$, $BC=a$, and $CA=b$. If $(a+b+c)(a+b-c)=ab$, what is the degree measure of $\angle C$?

   (A) 120 **(B)** 105 **(C)** 90 **(D)** 75 **(E)** 60

5. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

(A) 104 **(B)** 108 **(C)** 148 **(D)** 158 **(E)** 266

6. When expanded and simplified $(1 - x + x^2 - x^3)^{10}$ has the form

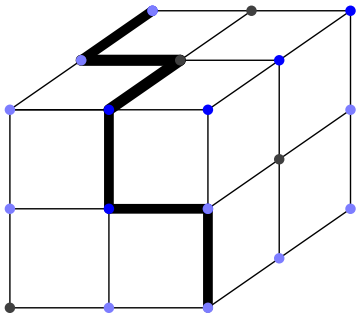
$$c_0 + c_1 x + c_2 x^2 + \cdots + c_{30} x^{30}$$

What is the value of $c_1 + c_3 + c_5 + \cdots + c_{29}$, the sum of the coefficients of all the odd powers of x?

(A) -4^{19} **(B)** -2^{20} **(C)** -2^{19} **(D)** 0 **(E)** 2^{19}

7. A small bug crawls on the surface of a $2 \times 2 \times 2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1 \times 1 \times 1$ cubes. How many shortest paths of this type are possible? One example is shown.

(A) 54 **(B)** 64 **(C)** 90 **(D)** 96 **(E)** 120



8. If $\sin x + \cos x = \sqrt{5}/3$, then what is the value of $\sqrt{\cos 4x}$?

(A) 5/9 **(B)** 7/9 **(C)** 7/11 **(D)** 8/11 **(E)** 12/13

9. Find the value of $r^2s^2 + s^2t^2 + t^2r^2$ if r, s, and t are the three possibly complex roots of the cubic polynomial $x^3 + 5x^2 - 3x + 1$.

(A) -1 **(B)** 0 **(C)** 3 **(D)** 5 **(E)** 8

10. What is the sum of all the odd 5 digit numbers in which each of the digits 1,2,3,4, and 5 occur exactly once?

(A) 2000976 **(B)** 2188876 **(C)** 2299936 **(D)** 2399976 **(E)** 2499936

11. If $a^2 - 2a + b^2 - 2b = ab - 4$, then what is the value of $a + 2b$?

(A) 0 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** cannot be uniquely determined

12. Consider the lines $y = 0$, $y = \sqrt{3}$, and $y = x\sqrt{3}$. Let C be the center of the circle tangent to all three lines such that the x-coordinate of this center is negative. The sum of the coordinates of C can be written in the form $a + b\sqrt{3}$ where a and b are rational numbers. Find $a + b$.

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

13. The graph of $z^2 = 4x^2 + 4y^2$ is a double cone and the graph of $2x - z = 8$ is a plane. The intersection of these two graphs is

(A) a circle **(B)** a non-circular ellipse **(C)** an hyperbola
(D) a parabola **(E)** two intersecting lines

14. What is the sum

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{8} + \frac{1}{10} + \frac{1}{16} + \frac{1}{20} + \frac{1}{25} + \cdots$$

which consists of the sum of the reciprocals of 1 and all the natural numbers whose prime factorizations contain no primes other than 2 or 5?

(A) 5/2 **(B)** 11/4 **(C)** 19/8 **(D)** 17/8 **(E)** 3

15. If $2 + \ln x = \ln(x+2)$ then x must equal

(A) $\frac{2}{e^2-1}$ **(B)** $\frac{2}{e-1}$ **(C)** $\frac{1}{e^2-1}$ **(D)** $\frac{2}{e^2+1}$ **(E)** $\frac{1}{e^2+1}$

16. In $\triangle ABC$, the medians AD and BE are perpendicular. If $AC = 8$ and $BC = 12$, what is the length of AB?

(A) 6 **(B)** $4\sqrt{13/5}$ **(C)** $4\sqrt{3}$ **(D)** 7 **(E)** 9

17. When the polynomial x^{2012} is divided by the polynomial $x^2 + x + 1$ what is the remainder $R(x)$?

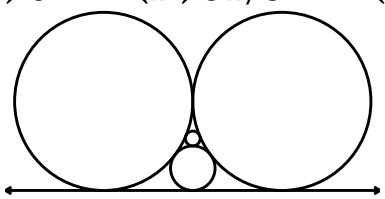
(A) 1 **(B)** $-x-1$ **(C)** $x+1$ **(D)** $2x-1$ **(E)** 0

18. A 2012×2012 matrix A has its entry in the ith row and jth column designated by $a_{i,j}$. Suppose for each $k = 1, 2, 3, \cdots, 1006$ that $a_{2k-1,2k-1} = 4k-3$, $a_{2k-1,2k} = 4k-2$, $a_{2k,2k-1} = 4k-1$, and $a_{2k,2k} = 4k$. All other values of $a_{i,j}$ are set equal to zero. Find the value of $\det A$, where $\det A$ stands for the determinant of A.

(A) -4^{1006} **(B)** -2^{1006} **(C)** 0 **(D)** 2^{1006} **(E)** 4^{1006}

19. Two congruent large circles and a smaller third circle are mutually externally tangent and also tangent to the same line, as shown. A fourth circle of diameter one, smaller than the rest, is drawn tangent to these three circles. What is the radius of the two large congruent circles?

(A) 5 **(B)** $4\sqrt{2}$ **(C)** 6 **(D)** 31/5 **(E)** 19/3



20. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

(A) 5/12 **(B)** 1/11 **(C)** 1/24 **(D)** 1/22 **(E)** 1/48