Spring 2013 McNabb GDCTM Contest
Algebra Two

NO Calculators Allowed

1. If 10% of a is b what is 10% of b?

   (A) $100a$ **(B)** $10a$ **(C)** a **(D)** $.1a$ **(E)** $.01a$

2. When a cyclist gets a puncture she has just completed three-fourths of her route. She finishes her route by walking. If she spent twice as much time walking as biking, how many times faster does she bike than walk?

   (A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

3. How many seconds are there in exactly six weeks?

   (A) 7! **(B)** 8! **(C)** 9! **(D)** 10! **(E)** 12!

4. If 10 carpenters can build 10 cabinets in 10 days how many days does it take 20 carpenters to build 20 cabinets?

   (A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

5. Which of the following equations has exactly two solutions over the real numbers?

   (A) $x^2 - 6x + 9 = 0$ **(B)** $5x = 2(5 - 7x)$ **(C)** $|x + 8| = -5$
   (D) $|x| = 12$ **(E)** $x^2 + 1 = 0$

6. In the following triangular arrangement of the positive integers, in which column, counting from left to right, does 7021 appear?

$$\begin{array}{ccccc} & & \cdot & & \\ & & 5 & \cdots & \\ & 2 & 6 & \cdots & \\ 1 & 3 & 7 & \cdots & \\ & 4 & 8 & \cdots & \\ & & 9 & \cdots & \\ & & \cdot & & \end{array}$$

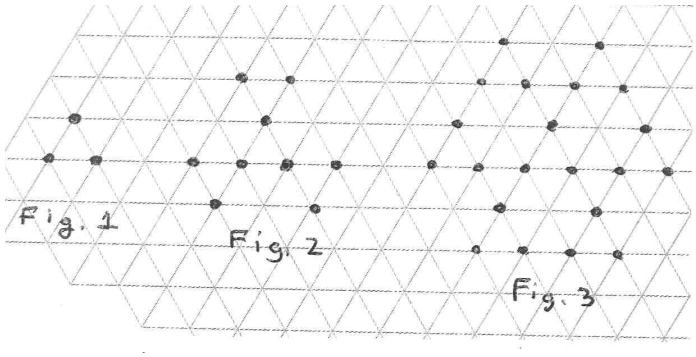
   (A) 43 **(B)** 51 **(C)** 52 **(D)** 84 **(E)** 99

7. The number of digits in the large number 2^{50} is

   (A) between 6 and 10 inclusive
   (B) between 11 and 15 inclusive
   (C) between 16 and 20 inclusive
   (D) between 21 and 25 inclusive
   (E) 26 or more

8. The first three figures of a certain sequence of figures are shown below on an equilateral triangle grid. Each successor figure is obtained recursively from its predeccesor by this rule: any two or more consecutive dots on a grid line generate new neighboring dots on that grid line, on either side, where no dot was before. All previous dots remain. How many dots does the 5th figure in this sequence have?

   (A) 34 **(B)** 36 **(C)** 57 **(D)** 59 **(E)** 64



9. For how many positive integers n does $n!$ end in exactly eleven zeros?

   (A) 0 **(B)** 3 **(C)** 5 **(D)** 8 **(E)** 11

10. Which of these numbers is the least?

    (A) $\log_8 144$ **(B)** $\log_4 72$ **(C)** $\log_{16} 288$ **(D)** $\log_2 48$ **(E)** $\log_{32} 576$

11. Quadrilateral $PQRS$ is inscribed in a circle. Segments PQ and SR are extended to meet at T. If $\angle SPQ = 80^\circ$ and $\angle PQR = 130^\circ$, find in degrees the measure of $\angle T$.

    (A) 50 **(B)** 53 **(C)** 57 **(D)** 60 **(E)** 61

12. If the equations $x^2 + ax + 21 = 0$ and $2x^2 + 19x + 35 = 0$ have a solution in common, what could be the value of the constant a?

(A) -10 **(B)** -4 **(C)** -2 **(D)** 4 **(E)** 10

13. Which transformation never changes the median of a list of a dozen distinct positive integers?

(A) adding 6 to each number in the list
(B) adding 3 to each of the three smallest numbers in the list
(C) subtracting 4 from each of the four largest numbers in the list
(D) doubling each number in the list
(E) taking the reciprocal of each number in the list

14. How many different paths are there from $(0,0)$ to $(4,4)$ if only these three kinds of steps may be taken: (i) a unit step to the right, (ii) a unit step up, (iii) a northeast diagonal step from point (i,j) to point $(i+1, j+1)$?

(A) 276 **(B)** 295 **(C)** 321 **(D)** 343 **(E)** 371

15. How many solutions in radians of $\sin 2\theta = \cos 3\theta$ lie in the interval $[0, 2\pi]$?

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

16. Let $f(x) = (1/4)x^2 + bx + c$ where b and c are constants. If b and c are chosen randomly and independently from the set of digits $\{0,1,2,3,4,5,6,7,8,9\}$ what is the probability that the vertex of the parabola $y = f(x)$ lies on the x-axis?

(A) $1/25$ **(B)** $1/20$ **(C)** $1/10$ **(D)** $4/25$ **(E)** $1/5$

17. Let a, b, and n be constants, with n a positive integer. If the first three terms of the binomial expansion of $(a+x)^n$ are, in ascending powers of x, equal to $3b + 6bx + 5bx^2$, then find the value of $a + b + n$.

(A) 48 **(B)** 64 **(C)** 96 **(D)** 128 **(E)** 252

18. When $x^{101} + x^{51} + 1$ is divided by $x^3 + 1$, what is the remainder?

(A) 0 **(B)** x **(C)** $3x^2 + 4x - 2$ **(D)** -1 **(E)** $-x^2$