Spring 2012 McNabb GDCTM Contest
Algebra II

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. The value of $\left(\sqrt{12}+\sqrt{3}\right)^2$ is

   (A) 36 **(B)** 27 **(C)** 21 **(D)** 15 **(E)** 12

2. A cage contains birds and rabbits. There are seventeen heads and forty feet. How many rabbits are in the cage?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

3. Mr. and Mrs. Reynolds have three daughters and three sons. At Easter each member of the family buys one chocolate Easter egg for everyone else in the family. How many Easter eggs will the Reynolds family buy in total?

   (A) 28 **(B)** 32 **(C)** 40 **(D)** 56 **(E)** 64

4. If $a \diamond b$ equals the lesser of $1/a$ and $1/b$, find the value of $-3 \diamond (-2 \diamond (-1/2))$.

   (A) -3 **(B)** -1/3 **(C)** -1/2 **(D)** -2 **(E)** -1

5. What is the y-intercept of the perpendicular bisector of the segment with endpoints $(-2, 8)$ and $(8, 4)$?

   (A) -1/2 **(B)** 1/2 **(C)** 0 **(D)** -3/2 **(E)** 3/2

6. The graph of $z^2 = 4x^2 + 4y^2$ is a double cone and the graph of $2x - z = 8$ is a plane. The intersection of these two graphs is

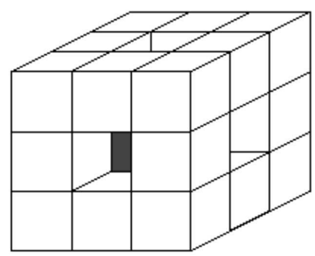
   (A) a circle **(B)** a non-circular ellipse **(C)** an hyperbola
   (D) a parabola **(E)** two intersecting lines

7. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

(A) 104 **(B)** 108 **(C)** 148 **(D)** 158 **(E)** 266

8. Twenty seven small $1 \times 1 \times 1$ cubes are glued together to form a $3 \times 3 \times 3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

(A) 56 **(B)** 64 **(C)** 72 **(D)** 84 **(E)** 96



9. If $2 + \ln x = \ln(x + 2)$ then x must equal

(A) $\dfrac{2}{e^2 - 1}$ **(B)** $\dfrac{2}{e - 1}$ **(C)** $\dfrac{1}{e^2 - 1}$ **(D)** $\dfrac{2}{e^2 + 1}$ **(E)** $\dfrac{1}{e^2 + 1}$

10. Hezy and Zeke are both painters. Working alone Hezy can paint a certain room in 4 hours. Working alone Zeke can paint the same room in 6 hours. Let r_H, r_Z, and r_{HZ} be respectively the rates at which Hezy works alone, Zeke works alone, and Hezy and Zeke work together. Suppose for some coefficient of efficiency k satisfying $0 \leq k \leq 1$, these rates are related by the formula

$$r_{HZ} = k(r_H + r_Z)$$

Very often $k < 1$ since the two working at the same time interfere with each other to some extent. What is the value of k if in fact it takes Hezy and Zeke working together 2.7 hours to paint this room?

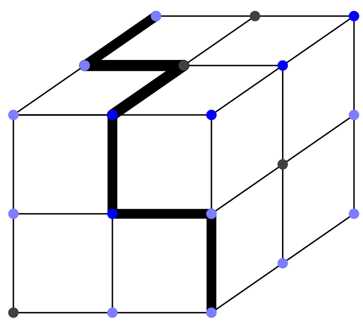
(A) 6/7 **(B)** 7/8 **(C)** 8/9 **(D)** 9/10 **(E)** 10/11

11. If $a^2 - 2a + b^2 - 2b = ab - 4$, then what is the value of $a + 2b$?

(A) 0 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** cannot be uniquely determined

12. A small bug crawls on the surface of a $2 \times 2 \times 2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1 \times 1 \times 1$ cubes. How many shortest paths of this type are possible? One example is shown.

(A) 54 **(B)** 64 **(C)** 90 **(D)** 96 **(E)** 120



13. Find the value of $r^2s^2 + s^2t^2 + t^2r^2$ if r, s, and t are the three possibly complex roots of the cubic polynomial $x^3 + 5x^2 - 3x + 1$.

(A) -1 **(B)** 0 **(C)** 3 **(D)** 5 **(E)** 8

14. In $\triangle ABC$ points D, E, and F lie on segments $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ respectively, in such a way that the proportions $BD/DC = 7/3$, $CE/EA = 3/2$, and $AF/FB = 4/1$ hold. If AD and FE intersect at G, what is the ratio AG/GD?

(A) $5/6$ **(B)** $6/7$ **(C)** $7/8$ **(D)** $8/9$ **(E)** $1/1$

15. If $x > \frac{1}{x}$, then which of the following must be true?

I. $2x > \frac{2}{x}$

II. $2x > \frac{1}{2x}$

III. $x^2 > \frac{1}{x^2}$

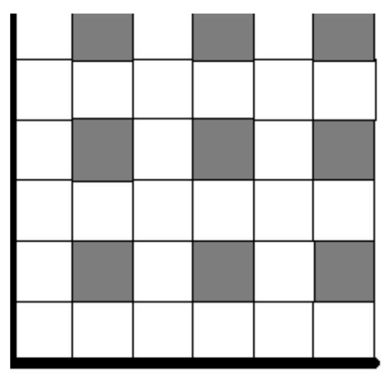
(A) I only **(B)** I and II only **(C)** I and III only **(D)** II and III only
(E) I, II, and III

16. When the polynomial x^{2012} is divided by the polynomial $x^2 + x + 1$ what is the remainder $R(x)$?

(A) -1 **(B)** $x + 1$ **(C)** $2x - 1$ **(D)** 0 **(E)** $-x - 1$

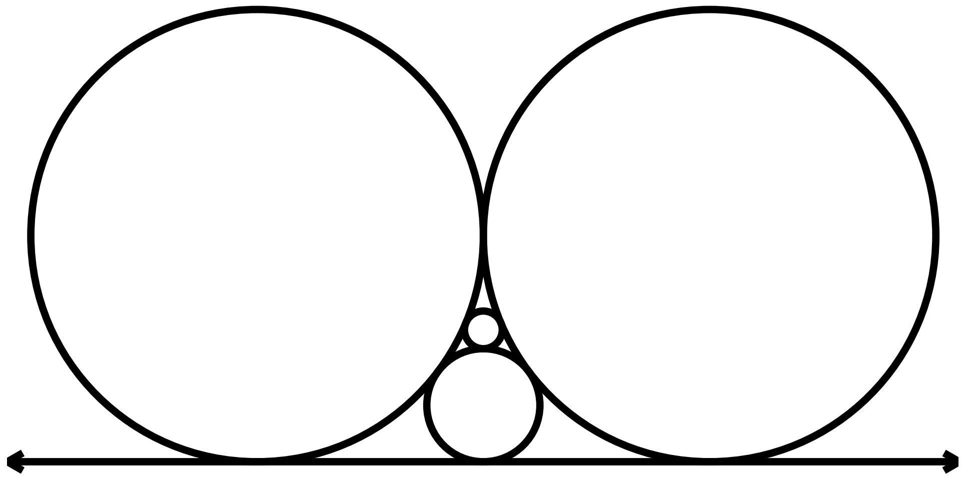
17. A rectangular lobby is to be tiled in this pattern in such a way that the border tiles along all the edges of the lobby are white. If the lobby measures 201 tiles by 101 tiles, how many shaded tiles are required?

(A) 4900 **(B)** 4950 **(C)** 5000 **(D)** 5050 **(E)** 5100



18. Two congruent large circles and a smaller third circle are mutually externally tangent and also tangent to the same line, as shown. A fourth circle of diameter one, smaller than the rest, is drawn tangent to these three circles. What is the radius of the two large congruent circles?

(A) $4\sqrt{2}$ **(B)** $31/5$ **(C)** 5 **(D)** $19/3$ **(E)** 6



19. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

(A) $5/12$ **(B)** $1/11$ **(C)** $1/24$ **(D)** $1/22$ **(E)** $1/48$

20. Let $\angle ABC$ measure 30 degrees. Imagine the rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are silvered as a mirror to reflect light. For a light beam that starts anywhere in the interior of $\angle ABC$, what is the maximum number of times such a beam can strike these mirrors?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8