Spring 2018 McNabb GDCTM Contest
Geometry

NO Calculators Allowed/ 60 Minutes

1. A certain triangle has an area of 200. If its height is doubled but its base kept the same, what is the area of the new triangle?

2. The points A, B, and C are collinear with $AB = 18$ and $BC = 32$. Find the sum of all possible values of AC.

3. What is the area of a triangle with side lengths 6, 6, and 4?

4. The positive integers are put in a rectangular grid in the following way

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 18 & 17 & 16 & 15 & 14 & 13 & 12 & 11 & 10 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}$$

   What is the number at the top of the column which contains the number 70?

5. Quadrilateral $ABCD$ has $AB = BC = 15$, $CD = 12$, and $DA = 9$. If $\angle B = 60^\circ$, find the area of $ABCD$.

6. A thin metal plate of uniform density has the shape of a quadrilateral with vertices at $(0, 0)$, $(6, 0)$, $(4, 4)$, and $(2, 4)$. Find the coordinates of the center of mass of this plate.

7. Solve:
$$|x - 5| - |3x + 5| + |2x + 10| = 4$$

8. Events A, B, and C are mutually independent with $P(A) = 0.2$, $P(B) = 0.3$, and $P(C) = 0.15$, find the probability of the event $A \cup B \cup C$.

9. Let a be a fixed positive real number. Find the area of the triangle formed by the three lines
$$\begin{aligned} y &= ax \\ y &= \frac{x}{a} \\ x + y &= a \end{aligned}$$
   in terms of a.

10. For what value of the positive parameter a does the triangle with vertices $(0, 0)$, $(a, 2\sqrt{a})$, and $(-2/a, 1/\sqrt{a})$ have the least possible area?

11. Three different colors are available to color the sides of a square. In how many different ways can this be done? Two ways are the same if one can be rotated into the other.

12. In trapezoid $ABCD$ with $AB \parallel CD$ and $AB/CD = 1/6$, draw diagonals AC and BD intersecting at point E. Find the ratio of the area of $ABCD$ to the area of ABE.

13. Factor $x^4 - 6x^3 + 9x^2 - 4$ into two quadratic polynomials with integer coefficients.

14. Draw the circle inscribed in the triangle with sides 3, 4, and 5. Then draw a second circle that is externally tangent to the first circle and is also tangent to the sides of length 3 and 5. Find the radius of this second circle.

15. In $\triangle ABC$, the angle bisectors of $\angle B$ and $\angle C$ meet at point Q. The line through Q parallel to AB meets side BC at S. The line through Q parallel to AC meets side BC at T. The line through Q parallel to BC meets sides AB and AC at points P and R respectively. If $AB = 4$, $BC = 9$, and $CA = 11$, find the ratio ST/PR.