

FALL 2014 McNABB GDCTM CONTEST
GEOMETRY

NO Calculators Allowed

1. Fiji apples cost \$4.68 for a half-dozen and 90 cents a piece. Gala apples cost \$5.39 for a half-dozen and 97 cents a piece. If Sarah buys 8 Fiji apples and 9 Gala apples with a \$20 bill, how much change should she receive?
2. Jack and Jill start walking toward each other. Initially they were 700 meters apart. Jack walks $\frac{4}{3}$ as fast as Jill. When they meet, how far is Jack from where Jill started?
3. A brigade of over a thousand men can line up in 13 rows of equal length with 4 soldiers left over and it can line up in 19 rows of equal length with 1 soldier left over. What is the smallest possible size of the brigade?
4. Write down in order from least to greatest (separate by commas) these irrational numbers:

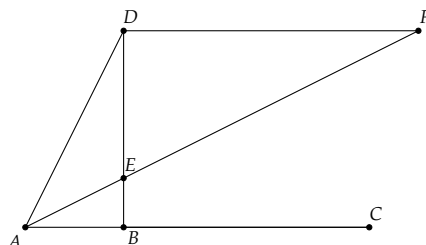
$$1 + \sqrt{3}, \quad 2 - \sqrt{2}, \quad 2\sqrt{2}, \quad \frac{\sqrt{2}}{2}$$

5. Solve:

$$|x - 3|x - 4|| = |7 + 2x|$$

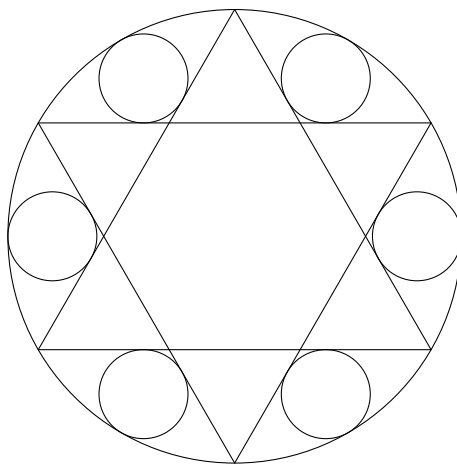
6. A fountain has two basins, one above and one below, each of which has three outlets. The first outlet of the top basin fills the lower basin in one hour, the second in two hours, and the third in three hours. When all three upper outlets are shut, the first outlet of the lower basin empties it in two hours, the second in three hours, and the third in four hours. If all the outlets are opened, how long in hours will it take for the lower basin to fill?
7. For m a positive integer, let $g(m)$ be the number of distinct prime factors of m . For example, $g(12) = 2$. Find the value of $g(g(60) \cdot g(91))$.
8. What is the first time after 1pm that the minute and hour hand of a clock will overlap? Answer to the nearest second and express your answer in hours:minutes:seconds format.

9. In the diagram, segment DB is perpendicular to both DF and AC , and $A - E - F$ are collinear. In addition, $EF = 2AD$. If $\angle FAC$ measures 17° find the angle measure in degrees of $\angle DAB$.



10. You are given three weighings involving twelve balls, of which eleven are the same weight but one is either heavier or lighter than the rest. The balls are numbered 1 through 12. The scale has two pans, a left and a right. When balls 1, 4, 7, 10 are put in the left pan and balls 3, 6, 9, 12 are put in the right pan, the left pan is heavier. When balls 3, 6, 9, 10 are put in the left pan and balls 2, 5, 8, 12 are put in the right pan, the left pan is lighter. When balls 3, 4, 8, 12 are put in the left pan and balls 2, 6, 7, 11 are put in the right pan, the right pan is heavier. Which ball is different and is it heavier or lighter than the rest?
11. The line $y = mx$ intersects the lines $x + y = 7$ and $x + y = -14$ at points A and B respectively. If $AB = 39$, what is a possible value for the slope m ?
12. In right $\triangle ABC$, its inscribed circle meets legs BA and BC at points D and E respectively. If $BD = 3$ and $DA = 11$, find the length of leg BC .
13. In $\triangle ABC$, point D lies on segment AB so that $AD/DB = 2/11$ while point E lies on segment BC so that $CE/EB = 3/7$. Let AE and CD intersect at point F . Find the ratio DF/FC .

14. A regular six-pointed star, composed of two intersecting equilateral triangles each of side length $6\sqrt{3}$, is inscribed in a circle. Six congruent smaller circles are internally tangent to this circle and externally tangent to the star. Find the radius of the small circles.



15. Find the volume of the tetrahedron with vertices located at $(0,0,0)$, $(1,-2,1)$, $(1,2,-1)$, and $(2,1,-1)$.