Spring 2012 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. A cage contains birds and rabbits. There are seventeen heads and forty feet. How many rabbits are in the cage?

   (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

2. Mr. and Mrs. Reynolds have three daughters and three sons. At Easter each member of the family buys one chocolate Easter egg for everyone else in the family. How many Easter eggs will the Reynolds family buy in total?

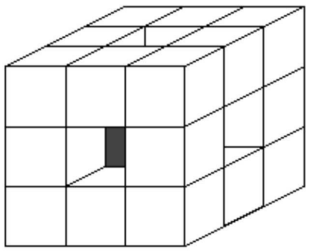
   (A) 28 (B) 32 (C) 40 (D) 56 (E) 64

3. Quadrilateral $ABCD$ has vertices in the coordinate plane as follows:
   $A = (0, 0), B = (5, 12), C = (-3, -3)$, and $D = (0, -7)$. The perimeter of this polygon equals

   (A) 40 (B) 42 (C) 45 (D) 46 (E) 48

4. Twenty seven small $1 \times 1 \times 1$ cubes are glued together to form a $3 \times 3 \times 3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

   (A) 56 (B) 64 (C) 72 (D) 84 (E) 96



5. What is the y-intercept of the perpendicular bisector of the segment with endpoints $(-2, 8)$ and $(8, 4)$?

   (A) -1/2 (B) 1/2 (C) 0 (D) -3/2 (E) 3/2

6. Three pairs of husbands and wives are to be seated at a bolted-down picnic table which seats exactly six people, three to a side. If no husband is to sit on the same side as his wife, and no wife is to sit directly across from her husband, in how many ways can these six persons be seated?

   (A) 24 (B) 60 (C) 64 (D) 80 (E) 96

7. What is the area of a triangle whose sides measure 13, 14, and 15?

   (A) 64 (B) 84 (C) 90 (D) 92 (E) 96

8. In rectangle $ABCD$ point P is located on side CD, closer to C than D, in such a way that $\angle APB$ is right. If $AB = 5$ and $AD = 2$, find the length of segment CP.

   (A) 1 (B) 6/5 (C) 5/4 (D) $\sqrt{2}$ (E) 2

9. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

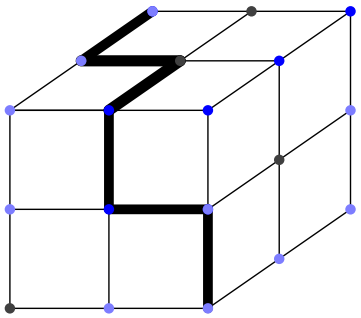
   (A) 104 (B) 108 (C) 148 (D) 158 (E) 266

10. In $\triangle ABC$ points D, E, and F lie on segments $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ respectively, in such a way that the proportions $BD/DC = 7/3$, $CE/EA = 3/2$, and $AF/FB = 4/1$ hold. If AD and FE intersect at G, what is the ratio AG/GD?

    (A) 5/6 (B) 6/7 (C) 7/8 (D) 8/9 (E) 1/1

11. A small bug crawls on the surface of a $2 \times 2 \times 2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1 \times 1 \times 1$ cubes. How many shortest paths of this type are possible? One example is shown.

    (A) 54 (B) 64 (C) 90 (D) 96 (E) 120



12. Hezy and Zeke are both painters. Working alone Hezy can paint a certain room in 4 hours. Working alone Zeke can paint the same room in 6 hours. Let r_H, r_Z, and r_{HZ} be respectively the rates at which Hezy works alone, Zeke works alone, and Hezy and Zeke work together. Suppose for some coefficient of efficiency k satisfying $0 \leq k \leq 1$, these rates are related by the formula

$$r_{HZ} = k(r_H + r_Z)$$

    Very often $k < 1$ since the two working at the same time interfere with each other to some extent. What is the value of k if in fact it takes Hezy and Zeke working together 2.7 hours to paint this room?

    (A) 6/7 (B) 7/8 (C) 8/9 (D) 9/10 (E) 10/11

13. In $\triangle ABC$, the medians AD and BE are perpendicular. If $AC = 8$ and $BC = 12$, what is the length of AB?

    (A) 6 (B) $4\sqrt{13/5}$ (C) 9 (D) $4\sqrt{3}$ (E) 7

14. Consider the lines $y = 0, y = \sqrt{3}$, and $y = \sqrt{3}x$. Let C be the center of the circle that is both tangent to all three of these lines and whose x-coordinate is negative. The sum of the coordinates for the center of C can written in the form $a + b\sqrt{3}$ where a and b are rational numbers. Determine $a + b$.

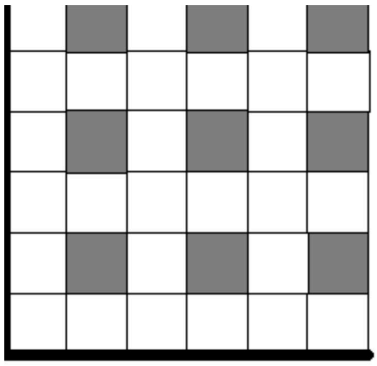
    (A) -1 (B) 0 (C) 1 (D) 2 (E) 3

15. A rectangular lobby is to be tiled in this pattern in such a way that the border tiles along all the edges of the lobby are white. If the lobby measures 201 tiles by 101 tiles, how many shaded tiles are required?

    (A) 4900 (B) 4950 (C) 5000 (D) 5050 (E) 5100

16. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

    (A) 5/12 (B) 1/11 (C) 1/24 (D) 1/22 (E) 1/48

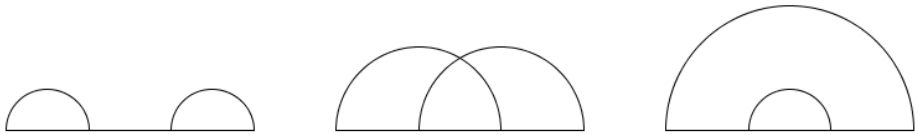


17. Let points A, B, C, and P lie in the $x-y$ plane. The coordinates of A, B, and C are $(0,0)$, $(3,0)$, and $(9,0)$ respectively. If $AP=7$ and $BP=6$, what is the value of CP?

(A) 8 (B) $\sqrt{65}$ (C) 25/3 (D) 9 (E) $\sqrt{85}$

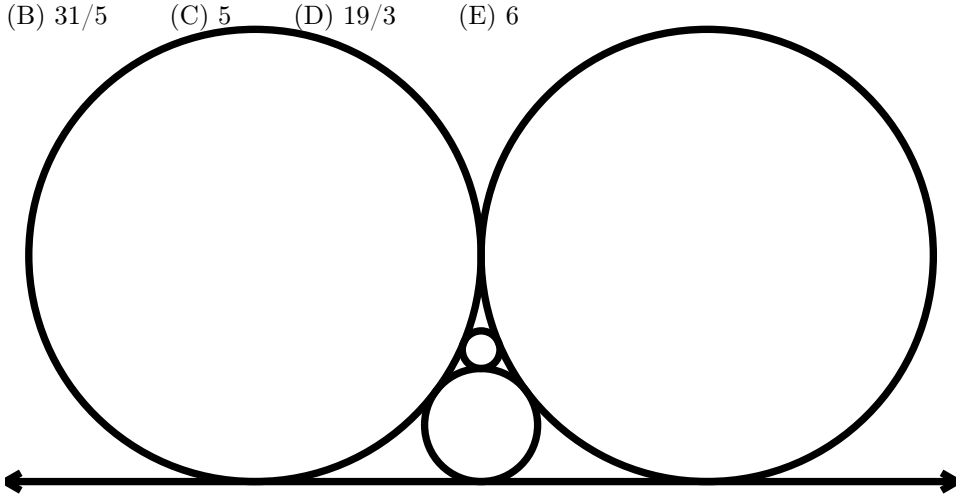
18. Distribute 14 points along a line segment. How many distinct ways are there for pairing these points using semicircles? The case of four points is pictured below.

(A) 10395 (B) 40320 (C) 60125 (D) 101245 (E) 135135



19. Two congruent large circles and a smaller third circle are mutually externally tangent and are also tangent to the same line, as shown. A fourth circle of diameter one, smaller than the rest, is drawn tangent to these three circles. What is the radius of the two large congruent circles?

(A) $4\sqrt{2}$ (B) 31/5 (C) 5 (D) 19/3 (E) 6



20. Let $\angle ABC$ measure 30 degrees. Imagine the rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are silvered as a mirror to reflect light. For a light beam that starts anywhere in the interior of $\angle ABC$, what is the maximum number of times such a beam can strike these mirrors?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7