Spring 2010 McNabb GDCTM Contest
Level III

1. Amy is in a line of 40 people at the movies. Ahead of her stand 12 people. How many stand behind her?

   (A) 13 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 39

2. Abigail, Brice, and Carl all start out with some one-dollar bills in their pockets. In particular, Carl starts out with 4 one-dollar bills. Brice then gives half of his dollar bills to Abigail and the other half to Carl. Then Abigail gives half of her bills to Brice and the other half to Carl. Finally, Carl gives half of his bills to Abigail and keeps the rest. If now, at the end of these exchanges, Abigail, Brice, and Carl all have 8 one-dollar bills, how many such bills did Abigail have to begin with?

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

3. Let a, b, c, and d be positive real numbers such that $abc = 12$, $bcd = 6$, $acd = 125$, and $abd = 3$. Then $abcd$ equals

   (A) 12 **(B)** 15 **(C)** 30 **(D)** 45 **(E)** 60

4. Let $f(x)$ be a linear function for which $f(8) - f(1) = 11$. Then $f(41) - f(6)$ equals

   (A) 37 **(B)** 43 **(C)** 49 **(D)** 55 **(E)** 61

5. Three pomegranates and one pineapple weigh as much as sixteen plums. Four plums and one pomegranate weight as much as one pineapple. How many pomegranates weigh as much as 3 pineapples?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

6. If p and q are integers and
$$p \log_{200} 5 + q \log_{200} 2 = 3$$
   then determine the value of $p + q$.

   (A) 10 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 20

7. For $z = a + bi$ a complex number, it's conjugate is $\overline{z} = a - bi$. Let S denote the set of all complex numbers z so that the real part of $1/\overline{z}$ equals one. Then set S is

   (A) a line **(B)** a circle **(C)** a parabola
   (D) the empty set **(E)** an hyperbola

8. If a is a multiple of 14 and b is a multiple of 21, then what is the largest integer that must be a factor of any integer of the form $9a + 8b$?

(A) 84 **(B)** 42 **(C)** 21 **(D)** 14 **(E)** 8

9. How many lines of symmetry does a cube have?

(A) 4 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 13

10. The parabola $y = ax^2 + bx + c$ passes through the points $(-2, 3)$, $(2, -1)$, and $(6, 12)$. The value of the coefficient a equals

(A) 1/4 **(B)** 3/16 **(C)** 5/16 **(D)** 17/32 **(E)** 1/2

11. The *centroid* of a triangle is the point of concurrence of its medians. In the $x - y$ plane point A has coordinates $(0, 0)$, point B has coordinates $(5, 15)$, and point C has coordinates $(13, 9)$. The line p passes through the point B and the centroid of $\triangle ABC$. Another point on line p is

(A) $(6, 9)$ **(B)** $(12, -2)$ **(C)** $(7, 1)$ **(D)** $(0, 43)$ **(E)** $(-4, -4)$

12. If $\sin x + \cos x = a$, then $\sin 2x$ equals

(A) $2a$ **(B)** $a^2 - 1$ **(C)** $1 - a^2$ **(D)** $a^2 + 1$ **(E)** $(a - 1)^2$

13. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

(A) $\frac{5}{72}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{16}$ **(E)** $\frac{3}{8}$

14. The function $f(x) = \dfrac{x + \sqrt{x^2 + 8}}{2}$ has domain $(-\infty, \infty)$ and on this domain is invertible. It's inverse function has the form $f^{-1}(x) = ax + \dfrac{b}{x}$ for some constants a and b. What is the value of $a + b$?

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

15. If $x + \dfrac{1}{x} = 3$ then what is the value of $\dfrac{x^4 + 1}{x^2}$?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

16. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(A) 576 **(B)** 864 **(C)** 1152 **(D)** 1200 **(E)** 1600

17. A semicircle lies in $\triangle EFG$ with diameter contained in $\overline{EG}$, and with $\overline{EF}$ and $\overline{GF}$ both tangent to it. If $EF = 12$, $FG = 15$, and $EG = 18$, what is the value of EC where C is the center of the semicircle?

(A) 6 **(B)** 6.5 **(C)** 7 **(D)** 7.5 **(E)** 8

18. A cubic polynomial $P(x)$ satisfies $P(1) = 1$, $P(2) = 3$, $P(3) = 5$, and $P(4) = 6$. Then the value $P(7)$ must equal

(A) 10 **(B)** 7 **(C)** 0 **(D)** -3 **(E)** -7

19. Hezy tosses a fair cubical die until each number 1 through 6 has come up at least once. On average, Hezy requires X tosses to do this. The real number X is closest to

(A) 12.8 **(B)** 14.7 **(C)** 16.3 **(D)** 17.2 **(E)** 19.5

20. In $\triangle ABC$, points D, E, and F are located on $\overline{BC}$, $\overline{AC}$, and $\overline{BA}$ respectively, so that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are concurrent at point P, the area of $\triangle BPD$ is 190, the area of $\triangle DPC$ is 380, and the area of $\triangle CPE$ is 418. Then the area of $\triangle APE$ is

(A) 121 **(B)** 143 **(C)** 242 **(D)** 319 **(E)** 330