Spring 2018 McNabb GDCTM Contest
Algebra One

NO Calculators Allowed/ 45 Minutes

1. A train passes a standing observer in seven seconds and a 378 yard platform station in 25 seconds. How long in yards is the train?

2. In how many ways can the letters in MCNABB be arranged such that the M and the N are never adjacent but the two B's are always adjacent?

3. A solution of 18 liters is 20% acid. How many liters of 68% acid must be added to the original solution to form a solution that is 36% acid?

4. In how many ways can 80 identical marbles be divided among eight children so that two of the children get 16 marbles each, four of them get 10 each, and two of them get 4 each?

5. How many integers n satisfy
$$\frac{15n}{13} - 8 < \frac{11n}{9} < \frac{13n}{11} + 8$$

6. Let a be a fixed positive real number. Find the area of the triangle formed by the three lines
$$\begin{aligned} y &= ax \\ y &= \frac{x}{a} \\ x + y &= a \end{aligned}$$
in terms of a.

7. Events A, B, and C are mutually independent with $P(A) = 0.2$, $P(B) = 0.3$, and $P(C) = 0.15$, find the probability of the event $A \cup B \cup C$.

8. A thin metal plate of uniform density has the shape of a quadrilateral with vertices at $(0, 0)$, $(6, 0)$, $(4, 4)$, and $(2, 4)$. Find the coordinates of the center of mass of this plate.

9. A biased coin is such that when tossed eight times the probability of getting exactly three heads is the same as the probability of getting exactly four heads. When this coin is tossed once, what is the probability of getting heads? Assume that the probability of getting heads when tossed once is neither zero nor one.

10. What is the fewest number of multiplications required to calculate a^{45}? The least efficient way is to multiply one a at a time for a total of 44 multiplications. A more efficient way is to multiply one a at a time to get to a^{15}. Then multiply three a^{15}'s together. This requires 16 multiplications altogether.

11. If r and s are the roots of the quadratic equation
$$2x^2 + 2x - 17 = 0$$
find the value of
$$r^2s^2 + rs^2 + sr^2 + rs + 1$$

12. After expanding and simplifying the product

$$a(a+b)(a+b+c)(a+b+c+d)(a+b+c+d+e)(a+b+c+d+e+f)$$

how many terms remain?

13. Solve:

$$|x-5|-|3x+5|+|2x+10|=4$$

14. Find one positive integer value of n so that 12^n-1 is divisible by 25.

15. For what value of the positive parameter a does the triangle with vertices $(0,0)$, $(a,2\sqrt{a})$, and $(-2/a,1/\sqrt{a})$ have the least possible area?