Fall 2014 McNabb GDCTM Contest
Algebra One

NO Calculators Allowed

1. Fiji apples cost \$4.68 for a half-dozen and 90 cents a piece. Gala apples cost \$5.39 for a half-dozen and 97 cents a piece. If Sarah buys 8 Fiji apples and 9 Gala apples with a \$20 bill, how much change should she receive?

2. If a ounces of tea leaves brews b cups of tea and c cups fill one thermos, how many ounces of tea leaves must be brewed to fill d thermos's? Answer in terms of a, b, c, and d.

3. A brigade of over a thousand men can line up in 13 rows of equal length with 4 soldiers left over and it can line up in 19 rows of equal length with 1 soldier left over. What is the smallest possible size of the brigade?

4. Suppose that m and n are positive integers satisfying

$$\frac{1}{77} = \frac{n}{7} - \frac{3}{m}$$

   Find the value of $m + n$.

5. Write down in order from least to greatest (separate by commas) these irrational numbers:

$$1+\sqrt{3}, \qquad 2-\sqrt{2}, \qquad 2\sqrt{2}, \qquad \frac{\sqrt{2}}{2}$$

6. Recall that $\binom{m}{n}$ stands for the number of ways of choosing n objects from a set of m objects. Name a solution m greater than 2 of the equation

$$\binom{m+2}{3} = 4\binom{m}{2}$$

7. Simplify

$$x - y - (3x - 4y) - (x - y - (3x - 4y))$$

8. How many arrangements of the letters in GDCTM do not have any 3 consecutive letters in alphabetical order? So, for instance, you would count DGCTM but you would not count DGCMT.

9. Solve:

$$\Big|x - 3|x - 4|\Big| = |7 + 2x|$$

10. Suppose n is a positive integer greater than or equal to 10. For such n, define $f(n)$ to be sum of the units digit of n and twice the value of $f(m)$ where m is the integer that remains when the units digit of n is removed. In case n is a positive integer less than or equal to 9, define $f(n) = n$. Find the value of $f(1145)$.

11. Simplify $\left(\sqrt{6}^{\sqrt{12}}\right)^{\sqrt{3}}$.

12. What is the first time after 1pm that the minute and hour hand of a clock will overlap? Answer to the nearest second and express your answer in hours:minutes:seconds format.

13. Find two positive rational numbers r and s, neither of which are integers, so that $r^2 + s^2 = 13$.

14. You are given three weighings involving twelve balls, of which eleven are the same weight but one is either heavier or lighter than the rest. The balls are numbered 1 through 12. The scale has two pans, a left and a right. When balls $1, 4, 7, 10$ are put in the left pan and balls $3, 6, 9, 12$ are put in the right pan, the left pan is heavier. When balls $3, 6, 9, 10$ are put in the left pan and balls $2, 5, 8, 12$ are put in the right pan, the left pan is lighter. When balls $3, 4, 8, 12$ are put in the left pan and balls $2, 6, 7, 11$ are put in the right pan, the right pan is heavier. Which ball is different and is it heavier or lighter than the rest?

15. The infinite expression
$$\sqrt{9+\sqrt{9+\sqrt{9+\cdots}}}$$
can be written in the form $\dfrac{a+\sqrt{b}}{c}$ where a, b, and c are positive integers with no common factor greater than one. Find the value of $a+b+c$.