Spring 2012 McNabb GDCTM Contest
Algebra One

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. The value of $-3 - 3^2 - 3^3$ is equal to

   (A) -40 **(B)** -39 **(C)** -37 **(D)** -36 **(E)** -33

2. A clever saleswoman is counting out envelopes for a customer. Every package of envelopes contains 80 envelopes. The saleswoman can count out 8 envelopes in 8 seconds. How many seconds does she need to count out 56 envelopes?

   (A) 24 **(B)** 48 **(C)** 56 **(D)** 72 **(E)** 80

3. If $a \diamond b$ equals the lesser of $1/a$ and $1/b$, find the value of $-3 \diamond (-2 \diamond (-1/2))$.

   (A) -3 **(B)** -1/2 **(C)** -1/3 **(D)** -2 **(E)** -1

4. What is the largest possible product of two positive odd integers whose sum is 40?

   (A) 39 **(B)** 279 **(C)** 300 **(D)** 399 **(E)** 400

5. Let
   $$S = \frac{1+2+4+8+16}{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}}$$
   Then S equals

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

6. A cage contains birds and rabbits. There are seventeen heads and forty feet. How many rabbits are in the cage?

   (A) 0 **(B)** 3 **(C)** 6 **(D)** 9 **(E)** 12

7. Mr. and Mrs. Reynolds have three daughters and three sons. At Easter each member of the family buys one chocolate Easter egg for everyone else in the family. How many Easter eggs will the Reynolds family buy in total?

   (A) 28 **(B)** 32 **(C)** 40 **(D)** 56 **(E)** 64

8. Three pairs of husbands and wives are to be seated at a bolted-down picnic table which seats exactly six people, three to a side. If no husband is to sit on the same side as his wife, and no wife is to sit directly across from her husband, in how many ways can these six persons be seated?

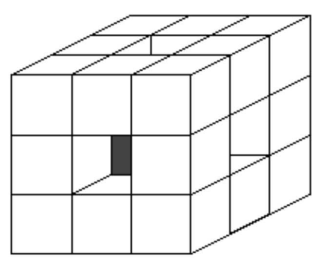
   (A) 24 **(B)** 60 **(C)** 64 **(D)** 80 **(E)** 96

9. Which integer below cannot be written as the sum of the squares of two integers?

   (A) 289 **(B)** 353 **(C)** 450 **(D)** 481 **(E)** 503

10. Twenty seven small $1 \times 1 \times 1$ cubes are glued together to form a $3 \times 3 \times 3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

    (A) 56 **(B)** 64 **(C)** 72 **(D)** 84 **(E)** 96



11. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B, who had not sold any yet. Then B sold 54 and had remaining half as many as A. How many cattle total did they have between them on their way to market?

    (A) 104 **(B)** 108 **(C)** 148 **(D)** 158 **(E)** 266

12. The value of $\left(\sqrt{12}+\sqrt{3}\right)^2$ is

(A) 40 **(B)** 36 **(C)** 30 **(D)** 27 **(E)** 24

13. Quadrilateral $ABCD$ has vertices in the coordinate plane as follows: $A=(0,0), B=(5,12), C=(-3,-3)$, and $D=(0,-7)$. The perimeter of this polygon equals

(A) 28 **(B)** 35 **(C)** 42 **(D)** 45 **(E)** 46

14. What is the remainder when the sum

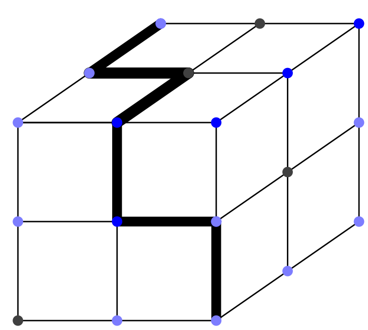
$$1^{111}+2^{111}+3^{111}+4^{111}+5^{111}+6^{111}+7^{111}+8^{111}+9^{111}+10^{111}$$

is divided by 11?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

15. A small bug crawls on the surface of a $2\times2\times2$ cube from one corner to the far opposite corner along the gridlines formed by viewing this cube as an assembly of eight $1\times1\times1$ cubes. How many shortest paths of this type are possible? One example is shown.

(A) 54 **(B)** 64 **(C)** 90 **(D)** 96 **(E)** 120



16. Given that $x \neq 0$, $y \neq 0$, and

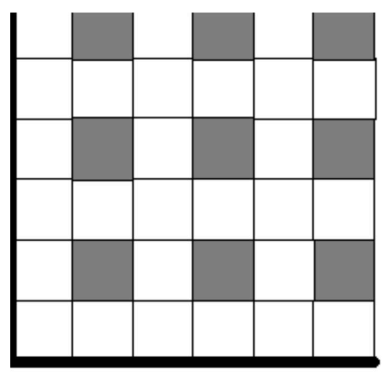
$$x = \frac{6}{y} - \frac{9}{xy^2}$$

what is the value of xy?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** cannot be uniquely determined

17. A rectangular lobby is to be tiled in this pattern in such a way that the border tiles along all the edges of the lobby are white. If the lobby measures 201 tiles by 101 tiles, how many shaded tiles are required?

(A) 4900 **(B)** 4950 **(C)** 5000 **(D)** 5050 **(E)** 5100

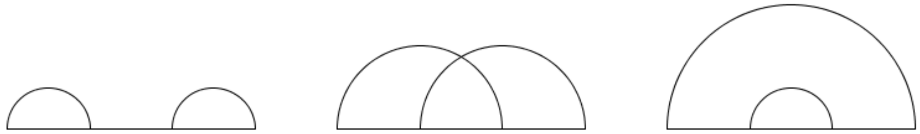


18. Jane took 5 tests, each time receiving a different score. These scores were all integers less than or equal to 100 and greater than or equal to zero. The average of her three lowest scoring tests was 84 while the average of her three highest scoring tests was 89. What is the maximum possible score of her highest scoring test?

(A) 94 **(B)** 95 **(C)** 96 **(D)** 97 **(E)** 98

19. In rectangle $ABCD$ point P is located on side CD, closer to C than D, in such a way that $\angle APB$ is right. If $AB = 5$ and $AD = 2$, find the length of segment CP.

(A) 5/7 **(B)** 6/7 **(C)** 1 **(D)** 2 **(E)** 3



20. Distribute 14 points along a line segment. How many distinct ways are there for pairing these points using semicircles? The case of four points is pictured above.

    (A) 10395 **(B)** 40320 **(C)** 60125 **(D)** 101245 **(E)** 135135

21. Three dice are rolled and it is known that the sum is a multiple of three. What is the probability that the sum is nine?

    (A) 1/3 **(B)** 25/72 **(C)** 13/48 **(D)** 1/2 **(E)** 11/18

22. For how many integer values of k can the polynomial $12x^2 + kx + 12$ be factored as $(ax + b)(cx + d)$ where a, b, c, and d are integers with $a \neq 0$ and $b \neq 0$?

    (A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

23. Find the coefficient of x in the expansion of

$$(x - 2012)(x - 2011)(x - 2010) \cdots (x + 2010)(x + 2011)(x + 2012)$$

    (A) $-(2012!)^2$ **(B)** $-(1006!)^2$ **(C)** 0 **(D)** $(1006!)^2$ **(E)** $(2012!)^2$

24. If $16^N = 2^1 \cdot 2^3 \cdot 2^6 \cdot 2^{10} \cdots 2^k$, where the exponents follow the triangular numbers, and k is the 20th triangular number, what is the value of N?

    (A) 190 **(B)** 215 **(C)** 300 **(D)** 385 **(E)** 420

25. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?

    (A) 5/12 **(B)** 1/11 **(C)** 1/24 **(D)** 1/22 **(E)** 1/48