Spring 2010 McNabb GDCTM Contest
Level II

1. What is the area of the quadrilateral in the coordinate plane with vertices whose coordinates are (in order): $(0,0)$, $(7,1)$, $(4,4)$, and $(2,11)$?

   (A) 30 **(B)** 31 **(C)** 31.5 **(D)** 32 **(E)** 33

2. Ronald has an unlimited number of 5 cent and 7 cent stamps. What is the largest amount of postage (in cents) that he cannot make with these stamps?

   (A) 16 **(B)** 22 **(C)** 23 **(D)** 79 **(E)** 99

3. Find the distance between the point with coordinates $(14,-2)$ and the line with equation $3x-4y=0$.

   (A) 4 **(B)** $4\sqrt{2}$ **(C)** $5\sqrt{2}$ **(D)** 8 **(E)** 10

4. Zeke cycles steadily for 36 miles. If he had managed to go 3 mph faster, he would have taken one hour less for the trip. What was Zeke's actual speed in mph during this trip?

   (A) 9 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 21

5. How many lines of symmetry does a cube have?

   (A) 4 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 13

6. In $\triangle ABC$, let D be the intersection point of the bisector of $\angle ABC$ and the bisector of $\angle BCA$. If $\angle CAB$ is 70 degrees, what is the measure of $\angle CDB$ in degrees?

   (A) 35 **(B)** 55 **(C)** 105 **(D)** 125 **(E)** 140

7. Hezy eats y yogurts every d days. How many yogurts does he eat in w weeks?

   (A) $\frac{7yw}{d}$ **(B)** $\frac{7w}{yd}$ **(C)** $\frac{yd}{7w}$ **(D)** $7dwy$ **(E)** $\frac{7yd}{w}$

8. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

   (A) $\frac{5}{72}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{16}$ **(E)** $\frac{3}{8}$

9. A set of seven distinct positive integers has a mean of 13. Find the difference between the greatest possible median of these integers and the least possible median of these integers.

   (A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

10. The parabola $y = ax^2 + bx + c$ passes through the points $(-2, 3)$, $(2, -1)$, and $(6, 12)$. The value of the coefficient a equals

(A) $1/4$ **(B)** $3/16$ **(C)** $5/16$ **(D)** $17/32$ **(E)** $1/2$

11. The *centroid* of a triangle is the point of concurrence of its medians. In the $x - y$ plane point A has coordinates $(0, 0)$, point B has coordinates $(5, 15)$, and point C has coordinates $(13, 9)$. The line p passes through the point B and the centroid of $\triangle ABC$. Another point on line p is

(A) $(6, 9)$ **(B)** $(12, -2)$ **(C)** $(7, 1)$ **(D)** $(0, 43)$ **(E)** $(-4, -4)$

12. If $\sin x + \cos x = a$, then $\sin 2x$ equals

(A) $2a$ **(B)** $a^2 - 1$ **(C)** $1 - a^2$ **(D)** $a^2 + 1$ **(E)** $(a - 1)^2$

13. A line L in the coordinate plane has slope -2. Suppose the triangle with vertices given by the origin, the x-intercept of L, and the y-intercept of L has area 9. Then an equation for L could be

(A) $2x + y = 0$ **(B)** $2x + y = 4$ **(C)** $-2x + y = 6$
(D) $2x + y = 3$ **(E)** $2x + y = -6$

14. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1 inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(A) 576 **(B)** 864 **(C)** 1152 **(D)** 1200 **(E)** 1600

15. In acute $\triangle ABC$, the altitude from A meets side $\overline{BC}$ at point D, the altitude from B meets side $\overline{AC}$ at point E, and the altitude from C meets side $\overline{AB}$ at point F. All three altitudes are concurrent at point H lying inside $\triangle ABC$. If $\angle BAC$ measures 58 degrees, then find the measure of $\angle BHC$ in degrees.

(A) 90 **(B)** 98 **(C)** 104 **(D)** 116 **(E)** 122

16. In how many ways can five distinct books be arranged in a bookcase with 3 shelves, each shelf capable of holding all five books?

(A) 19 **(B)** 120 **(C)** 360 **(D)** 840 **(E)** 2520

17. Two ferries start at the same instant from opposite banks of a river. They travel directly across the river. Each boat keeps its own constant speed, though one boat is faster than the other. In this first trip across they pass at a point 720 yards from the nearer bank. When reaching the opposite shore each boat remains exactly 10 minutes in its dock before heading back the other way. On this trip back the boats meet 400 yards from the other shore. How wide is the river (in yards)?

(A) 1040 **(B)** 1120 **(C)** 1520 **(D)** 1600 **(E)** 1760

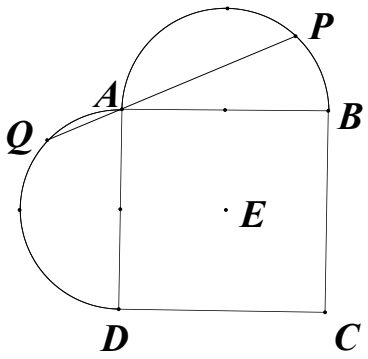
18. If p and q are integers and

$$p \log_{200} 5 + q \log_{200} 2 = 3$$

then determine the value of $p + q$.

(A) 10 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 20

19. Semicircles are drawn on two sides of square $ABCD$ as shown. Point E is the center of the square, and points Q, A, and P are collinear with $QA = 4$ and $AP = 16$. Find QE.



(A) 12 **(B)** $10\sqrt{2}$ **(C)** $10\sqrt{3}$
(D) 15 **(E)** 20

20. In $\triangle ABC$, points D, E, and F are located on $\overline{BC}$, $\overline{AC}$, and $\overline{BA}$ respectively, so that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are concurrent at point P, the area of $\triangle BPD$ is 190, the area of $\triangle DPC$ is 380, and the area of $\triangle CPE$ is 418. Then the area of $\triangle APE$ is

(A) 121 **(B)** 143 **(C)** 242 **(D)** 319 **(E)** 330