

SPRING 2012 McNABB GDCTM CONTEST

PREALGEBRA

NO Calculators Allowed

Note: all variables represent real numbers unless otherwise stated in the problem itself.

1. How many of the first 20 natural numbers, that is, the set $\{1, 2, 3, \dots, 20\}$, are composite? Recall that 1 is neither prime nor composite.
(A) 10 (B) 11 (C) 12 (D) 13 (E) 14
2. The value of $-3 - 3^2 - 3^3$ is equal to
(A) -41 (B) -40 (C) -39 (D) -37 (E) -36
3. A clever saleswoman is counting out envelopes for a customer. Every package of envelopes contains 80 envelopes. The saleswoman can count out 8 envelopes in 8 seconds. How many seconds does she need to count out 56 envelopes?
(A) 24 (B) 48 (C) 56 (D) 72 (E) 80
4. A jar contains only blue and green marbles in the ratio of 3 blue for every 5 green. If 3 green marbles are removed and replaced by 3 blue marbles, the ratio of blue to green marbles becomes one to one. How many green marbles were in the jar originally?
(A) 15 (B) 18 (C) 21 (D) 24 (E) 27
5. What is the largest possible product of two positive odd integers whose sum is 40?
(A) 39 (B) 279 (C) 300 (D) 399 (E) 400

6. What is the remainder when the sum

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3$$

is divided by 9?

- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

7. Before a grape is dried to become a raisin it is 94% water, while a raisin is only 25% water. If only water is evaporated from the grapes, how many kilograms of raisins can be made from 60 kilograms of grapes?

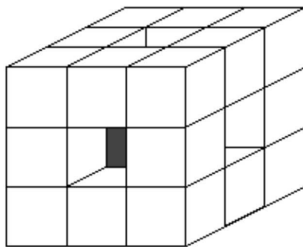
- (A) 4.8 (B) 18.2 (C) 24.6 (D) 27.6 (E) 45

8. Mr. and Mrs. Reynolds have three daughters and three sons. At Easter each member of the family buys one chocolate Easter egg for everybody else in the family. How many Easter eggs will the Reynolds family buy in total?

- (A) 28 (B) 32 (C) 40 (D) 56 (E) 64

9. Twenty seven small $1 \times 1 \times 1$ cubes are glued together to form a $3 \times 3 \times 3$ cube. Then the center small cube and the small cubes at the center of each face are removed. What is the surface area of the resulting solid?

- (A) 56 (B) 64 (C) 72 (D) 84 (E) 96



10. A train 2700 meters long passes a signal in 135 seconds. At that rate, how many seconds does it take to cross a bridge 1000 meters in length?

- (A) 50 (B) 110 (C) 135
(D) 160 (E) 185

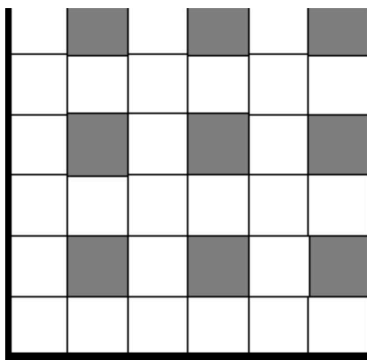
11. Which integer below cannot be written as the sum of the squares of two integers?
- (A) 289 (B) 353 (C) 450 (D) 481 (E) 503
12. The digits 7,5,3,2, and 0 are each used once to make the smallest possible 5 digit number divisible by 11. What is the hundred's digit of this number?
- (A) 0 (B) 2 (C) 3 (D) 5 (E) 7
13. A cylindrical can of juice holds 12 ounces. If the diameter of the can were to be doubled and the height halved, how many ounces would the new can hold?
- (A) 6 (B) 12 (C) 24 (D) 48 (E) 96
14. If the area of a circle is increased by 300% by what percent is the circumference increased?
- (A) 50% (B) 100% (C) 150% (D) 200% (E) 300%
15. Two drovers A and B went to market with cattle. A sold 50 and then had left as many as B , who had not sold any yet. Then B sold 54 and had remaining half as many as A . How many cattle total did they have between them on their way to market?
- (A) 104 (B) 108 (C) 148 (D) 158 (E) 266
16. A cage contains birds and rabbits. There are seventeen heads and forty feet. How many rabbits are in the cage?
- (A) 3 (B) 6 (C) 9 (D) 12 (E) 15
17. Water flows continuously into a 200ℓ tank at the rate of $3\ell/\text{min}$ and flows continuously out at the rate of $2\ell/\text{min}$. If the tank is initially 25% full, how many minutes will it take for the tank to fill completely?
- (A) 100 (B) 150 (C) 175 (D) 200 (E) 225

18. Jane took 5 tests, each time receiving a different score. These scores were all integers less than or equal to 100 and greater than or equal to zero. The average of her three lowest scoring tests was 84 while the average of her three highest scoring tests was 89. What is the maximum possible score of her highest scoring test?

(A) 95 (B) 96 (C) 97 (D) 98 (E) 99

19. A rectangular lobby is to be tiled in this pattern in such a way that the border tiles along all the edges of the lobby are white. If the lobby measures 201 tiles by 101 tiles, how many shaded tiles are required?

(A) 4900 (B) 4950 (C) 5000 (D) 5050 (E) 5100



20. The value of $(\sqrt{12} + \sqrt{3})^2$ is

(A) 40 (B) 36 (C) 27 (D) 21 (E) 15

21. Quadrilateral $ABCD$ has vertices in the coordinate plane as follows: $A = (0,0)$, $B = (5,12)$, $C = (-3,-3)$, and $D = (0,-7)$. The perimeter of this polygon equals

(A) 28 (B) 35 (C) 42 (D) 45 (E) 46

22. Let

$$S = \frac{1 + 2 + 4 + 8 + 16}{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}}$$

Then S equals

(A) 1 (B) 2 (C) 4 (D) 8 (E) 16

23. Three pairs of husbands and wives are to be seated at a bolted-down picnic table which seats exactly six people, three to a side. If no husband is to sit on the same side as his wife, and no wife is to sit directly across from her husband, in how many ways can these six persons be seated?
- (A) 24 (B) 60 (C) 64 (D) 80 (E) 96
24. For how many positive integers n is the least common multiple of 30, 24, and n equal to 600?
- (A) 8 (B) 9 (C) 10 (D) 12 (E) 18
25. A large circular metal plate has 12 equal smaller circular holes drilled out along its periphery to hold test tubes. Currently the plate holds no test tubes, but soon a robot arm will randomly place 5 test tubes on the plate. What is the probability that after all 5 of these test tubes are placed no two test tubes will be adjacent to one another?
- (A) $5/12$ (B) $1/11$ (C) $1/24$ (D) $1/22$ (E) $1/48$