Fall 2010 McNabb GDCTM Contest
Pre-Algebra

NO Calculators Allowed

1. Zeke's piggy bank has 111 coins. If it contains an equal number of quarters, dimes and nickels, and no other type of coin, the total value of the coins in his piggy bank is

   (A) \$14.80 **(B)** \$20.40 **(C)** \$24.00 **(D)** \$27.00 **(E)** \$44.40

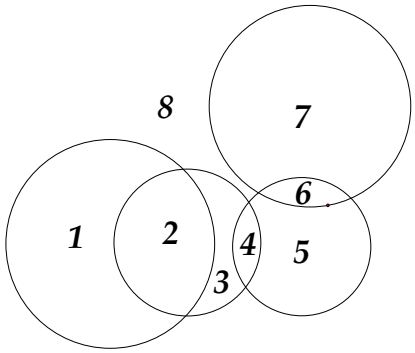
2. Keith's grandfather paid 70% of the cost of a jacket. Keith paid the rest. If Keith paid \$39 how much did his grandfather pay?

   (A) \$70 **(B)** \$91 **(C)** \$113 **(D)** \$130 **(E)** \$140

3. The remainder when 5^{2010} is divided by 7 equals

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 6

4. Four circles are drawn, all in the same plane. Find the maximum number of regions they can form. The diagram shows how four circles may form 8 regions.



   (A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

5. If the square root of a positive number falls between seven and eight, then the cube root of this number must fall between

   (A) 7 and 8 **(B)** 6 and 7 **(C)** 5 and 6
   (D) 4 and 5 **(E)** 3 and 4

6. What is the largest prime number which is a factor of every six digit number of the form $\underline{ABABAB}$?

   (A) 3 **(B)** 7 **(C)** 37 **(D)** 41 **(E)** 101

7. John has 54 coins totaling one dollar in value. Some are pennies; some are nickels; some are dimes. He has no other kind of coin. How many nickels does John have?

   (A) 7 **(B)** 9 **(C)** 11 **(D)** 13 **(E)** 15

8. How many lines of symmetry does a regular octagon have?

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

9. Seven consecutive integers are written on a whiteboard. When one of them is erased, the sum of the remaining six integers is 4208. What is the sum of the seven integers?

   (A) 4893 **(B)** 4900 **(C)** 4907 **(D)** 4914 **(E)** 4921

10. The next three Ranger batters get a hit with probabilities equal to 0.250, 0.300, and 0.500 respectively. What is the probability that all three get hits?

    (A) 1.000 **(B)** .300 **(C)** .125 **(D)** .05 **(E)** .0375

11. A positive integer has the interesting property that when expressed as a three digit base-7 number, those digits are the reverse of its digits when expressed as a base-9 number. What is this number expressed in normal form as a base-10 number?

    (A) 124 **(B)** 241 **(C)** 248 **(D)** 428 **(E)** 503

12. On a certain island, there are currently 1000 inhabitants, and 91% of these inhabitants were born there. Then some of these native inhabitants leave, so that now only 90% of the inhabitants of the island were born there. Assuming no other kind of change (births, deaths, immigration, etc...) in the population took place, how many of the native inhabitants left?

    (A) 9 **(B)** 10 **(C)** 40 **(D)** 90 **(E)** 100

13. How many positive integers less than 200 have an odd number of factors?

    (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 14

14. In how many ways can the the letters in *syzygy* be arranged so that the three y's do not all occur together?

    (A) 96 **(B)** 112 **(C)** 113 **(D)** 114 **(E)** 120

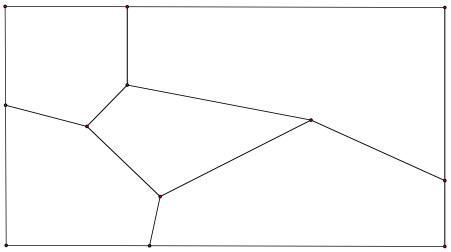
15. What is the greatest possible area of a triangle if two of its sides measure 8 and 13?

    (A) 41 **(B)** 47 **(C)** 48 **(D)** 51 **(E)** 52

16. For her Math Club's fundraiser, Carla biked 8 miles south, then 8 miles west, and finally 7 miles south. How many miles, as the crow flies, was she from her start point?

    (A) 14 **(B)** 15 **(C)** 17 **(D)** 20 **(E)** 23

17. A map maker has four colors available to color this map consisting of 5 counties. Each county is colored with a single color. No two counties that share a common boundary may be colored the same. In how many ways can our map maker color this map?



    (A) 48 **(B)** 60 **(C)** 72 **(D)** 96 **(E)** 120

18. Points A, B, C, and D are collinear and occur in the same order as given. If the ratio $AB : BC$ equals 3 and the ratio $BD : AB$ equals $8/3$, then determine the ratio $CD : AC$.

    (A) $\frac{5}{3}$ **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{7}{4}$ **(E)** $\frac{8}{3}$

19. What is the smallest possible product of three distinct numbers chosen from the set $\{-3, -2, -1, 0, 1, 3\}$

    (A) -18 **(B)** -9 **(C)** -6 **(D)** 0 **(E)** 2

20. The height of a square pyramid is increased by 60% and the sides of its base are decreased by 20%. By what percent is the volume of the pyramid increased?

(A) 40% **(B)** 4.8% **(C)** 4% **(D)** 2.4% **(E)** 0%

21. A baseball team has won 50 games out of 75 so far played. If there are 45 games yet to be played, how many of these must be won in order for the team to finish its season having won exactly 60% of its games?

(A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

22. How many factors of 720 are also factors of 630?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

23. The value of the fraction

$$\frac{3+6+9+\cdots+99}{4+8+12+\cdots+132}$$

is

(A) $2/3$ **(B)** $3/4$ **(C)** $4/5$ **(D)** $5/6$ **(E)** 7

24. To specify the order of operations in multiplying 5 numbers together, three sets of parentheses are needed. Two ways, for example, are $((ab)(cd))e$ and $(((ab)c)d)e$. In how many ways can these three sets of parentheses be arranged? Assume the order of the numbers a through e is never changed.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

25. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

(A) 32 **(B)** 28 **(C)** 24 **(D)** 20 **(E)** 16